WORKED EXAMPLES IN PHYSICS

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By

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Numbers in heavy type refer to problems, the others to pages

PREFACE TO THE SEVENTH EDITION

An attempt has been made to remedy deficiencies in the previous editions by introducing additional examples. It is intended that the range of subject matter dealt with should cover the syllabus of the examination for the General Certificate of Education at Advanced Level.

L. J. F.

1955.

PREFACE

This collection of worked examples is intended to be used to supplement a course of lectures or reading in Physics up to Intermediate or First M.B. standard. The solutions are not, of course, meant just to be read through; the student should work each problem for himself, comparing his own solution, when complete, with the one given in the book. To avoid unnecessary repetition, a table of formulæ and laws has been given, together with brief explanations of the symbols used. The student is advised to consult the appropriate section of this table before attempting the solution of the problems, so as to refresh his memory of the necessary formulæ.

In the problems on lenses, it has been thought advisable to provide two sets of solutions, one for each of the two sign conventions in common use. This is explained further in a note on p. 21.

Valuable assistance in the preparation of the book and in checking the solutions has been given by Dr. S. E. Green, to whom the author is deeply grateful.

L. J. F.

INTRODUCTION

THE solution of numerical problems in Physics is the best means of fixing the essentials of the subject in the student's mind. It is the nearest approach to a "royal road" for attaining a thorough grasp of the various principles and laws which form the basis of the study of Physics.

Time will be saved if a methodical approach to the solution of a problem is cultivated, in some such manner as the following:

(*a*) The problem should be read twice; the first time rapidly, to obtain an idea of the particular branch of the subject concerned; the second time slowly and carefully, so that the details have time to sink in.

(*b*) Next, there should be a mental review of the relevant knowledge possessed by the student, and a selection made of the laws, probably embodied in formulæ or equations, which will be needed in solving the problem (the table on p. 13 may be of help here). These formulæ may then be jotted down and the problem perused again to decide which of the various quantities are known numerically and which are as yet unknown. (Generally, it will be necessary to have as many equations as unknowns for a complete solution.)

(*c*) In some branches of the subject, notably Mechanics and Light, a diagram will greatly help in marshalling the data for inspection at a glance.

To illustrate the above suggestions, let us examine the following problem:

"How long will it take a wooden block to slide from rest a distance of 3 metres down a slope inclined at 30° to the horizontal, if the coefficient of friction between the block and the surface of the slope is 0·20?"

(*a*) The problem deals with friction, accelerated motion and the inclined plane.

(*b*) Some or all of the equations of motion will be required:

$$v=u+at$$

$$s=ut+\frac{1}{2}at^2$$

$$v^2=u^2+2as$$

$$F=ma$$

and also the expression for limiting friction:

$$f=\mu R$$

Of the quantities in these equations the following are given or known:

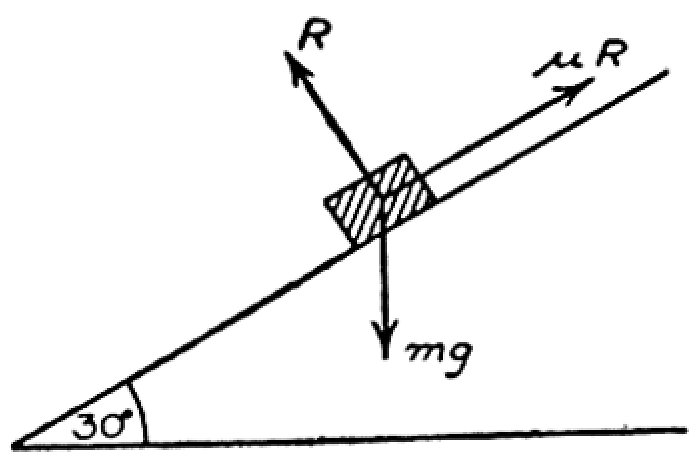
$s=300$ cms.
$u=0$ ("from rest")
$\mu=0{\cdot}20$

and we require to know t.

Each of the first two equations involves t; the first contains two additional unknowns, v and a; the second, only one, a. By selecting the second and fourth equations, we have only two unknowns, a and t, provided F, the accelerating force, can be determined in terms of m. This may be done by resolving all the forces acting on the block in a direction parallel to the plane, using the fifth equation to evaluate the limiting friction.

It therefore appears that the second, fourth and fifth equations are needed:

$$s=ut+\frac{1}{2}at^2$$
$$F=ma$$
$$f=\mu R$$



(c) In the diagram all the forces acting on the block must be shown:

mg=the weight of the block vertically down.
R=the normal reaction of the plane.
μR=the limiting friction up, parallel to the plane.

We have considered here only the approach to the solution of the problem, and it is in this approach that the student exercises his knowledge of physics. The solution proper will generally be an exercise in mathematics—algebra, geometry, trigonometry and arithmetic. (*See* Ex. 18.)

Most of the problems in this book have been worked with four-figure logarithms; in many cases sufficient accuracy could be obtained with a 10-in. slide-rule. But there are certain exceptions, for instance in expansion problems (Ex. 62), where the answer appears as a small difference between two quantities, which must themselves be taken to a higher degree of relative accuracy than the answer. Suppose, for example, we have to evaluate $(47{\cdot}73\times0{\cdot}9994)-(47{\cdot}61\times0{\cdot}9998)$ to 4 significant figures. This cannot be worked accurately with four-figure logarithms, but it can quickly be worked by simple arithmetic by writing it as follows:

$$\begin{aligned}
&47{\cdot}73(1-0{\cdot}0006)-47{\cdot}61(1-0{\cdot}0002)\\
&=(47{\cdot}73-0{\cdot}028638)-(47{\cdot}61-0{\cdot}009522)\\
&=(47{\cdot}73-47{\cdot}61)-(0{\cdot}028638-0{\cdot}009522)\\
&=0{\cdot}12-0{\cdot}019116\\
&=0{\cdot}1009 \text{ (to 4 significant figures)}
\end{aligned}$$

In certain types of calculation, it is desirable to make approximations in order to avoid unnecessarily long working; for example,

$$(1+a)^n=(1+na) \text{ approximately if } a \text{ is small.}$$

Here we have expanded $(1+a)^n$ by the binomial theorem and neglected terms in a^2, a^3, etc., as being negligibly small. Let us see to what extent this is justifiable.

$$1{\cdot}0003^2=\begin{cases}1{\cdot}0006 \text{ approx.}\\ 1{\cdot}00060009 \text{ exactly}\end{cases}$$

$$1{\cdot}003^2=\begin{cases}1{\cdot}006 \text{ approx.}\\ 1{\cdot}006009 \text{ exactly}\end{cases}$$

$$1{\cdot}03^2=\begin{cases}1{\cdot}06 \text{ approx.}\\ 1{\cdot}0609 \text{ exactly}\end{cases}$$

$$1{\cdot}3^2=\begin{cases}1{\cdot}6 \text{ approx.}\\ 1{\cdot}69 \text{ exactly}\end{cases}$$

The first of these approximations would be sufficiently accurate

for any type of practical problem, the last, with 6 per cent. error, for only the roughest calculation. The second is permissible where three- or four-figure accuracy is desired, and the third may be used with discretion for "slide-rule" accuracy.

Other forms of this approximation are:

$$\frac{1}{(1+a)^n}=(1+a)^{-n}=(1-na) \text{ approx. if } a \text{ is small}$$

and $$\frac{1+b}{1+a}=(1+b)(1-a)=(1+b-a) \text{ approx. if } a \text{ and } b \text{ are small.}$$

TABLE OF FORMULÆ AND LAWS

GENERAL PHYSICS

Mechanics		
$P=F\cos\theta$	P F θ	P the component of force F in a direction making an angle θ with the line of action of F
$v=u+at$	u v t a	Initial velocity. Velocity after time t. Time to attain velocity v. Acceleration.
$s=ut+\frac{1}{2}at^2$	s	Distance travelled in time t.
$v^2=u^2+2as$		
$F=ma$	F m	Force producing acceleration a in mass m. Mass of body being accelerated.
$a=\frac{v^2}{R}$	a R	Radial acceleration for a body travelling with velocity v along a circular path. Radius of path.
$\frac{1}{2}mv^2$=Kinetic energy mgh=Potential energy	g h	Acceleration due to gravity $=32{\cdot}2$ ft. per sec.^{2} or 981 cms. per sec.^{2} Height above ground.
mv=Momentum μR=Limiting friction	μ R	Coefficient of friction. Normal reaction between surfaces.
$p=\rho gh$	p h ρ	Pressure due to a column of liquid. Height of column of liquid. Density of liquid.

Archimedes' Principle: "The loss in weight of a body immersed in a fluid is equal to the weight of fluid displaced by the body."

Pendulum

$T=2\pi\sqrt{\frac{L}{g}}$	T L	Period of a simple pendulum. Length of pendulum.

Elasticity		
$Y=\frac{T}{a}\cdot\frac{L}{e}$	Y	Young's modulus of elasticity.
	L	Length of wire.
	a	Cross-sectional area of wire.
	T	Tension in wire.
	e	Extension due to tension T.
Surface Tension		
$h=\frac{2T}{r\rho g}$	h	Rise of liquid in capillary tube.
	r	Radius of tube.
	T	Surface tension of liquid.
	ρ	Density of liquid.
$p=\frac{2T}{r}$	p	Excess pressure in a bubble in a liquid.
	r	Radius of bubble.
$p=\frac{4T}{r}$	p	Excess pressure in a soap bubble (film with two surfaces).
Gas Laws—under HEAT.		

HEAT

Thermometry		
5 C. deg.$=$9 F. deg.		
$t°$ C.$=\left(\frac{9}{5}t+32\right)°$ F.		
$t°$ F.$=\frac{5}{9}(t-32)°$ C.		
Expansion		
$L_t=L_0\ (1+at)$	L_0	Length of body at 0° C.
	L_t	Length of a body at $t°$ C.
	t	Temperature.
	a	Coefficient of linear expansion.
$V_t=V_0\ (1+at)$	V_0	Volume of solid, liquid or gas at 0° C.
	V_t	Volume of solid, liquid or gas at $t°$ C.
	a	Coefficient of cubical expansion.
$\rho_0=\rho_t\ (1+at)$	ρ_0	Density of a substance at 0° C.
	ρ_t	Density of a substance at $t°$ C.

GAS LAWS

Boyle's Law pv=constant at constant temperature	p v	Pressure exerted by gas. Volume of given mass of gas.
Adiabatic Law pv^{γ}=constant	γ	Ratio of specific heats of a gas $=\frac{C_p}{C_v}$.

Charles's Law

$v_t=v_0\left(1+\frac{t}{273}\right)$ at constant pressure.

Pressure Law

$p_t=p_0\left(1+\frac{t}{273}\right)$ at constant volume.

General Gas Equation $pv=RT$	T	Absolute temperature $=t+273$.
	R	Gas constant.
Thermal Conductivity $\frac{Q}{t}=\frac{kA(\theta_1-\theta_2)}{d}$	Q	Quantity of heat conducted in time t.
	t	Time.
	k	Coefficient of thermal conductivity.
	A	Area across which heat is conducted.
	θ_1, θ_2	Temperatures of faces between which heat passes.
	$\frac{(\theta_1-\theta_2)}{d}$	Temperature gradient.

Newton's Law of Cooling: "The rate of cooling of a hot body is proportional to its excess temperature above its surroundings."

LIGHT

Snell's Law $\mu=\frac{\sin i}{\sin r}$	μ	Refractive index.
	i	Angle of incidence.
	r	Angle of refraction.

Mirrors		
$\frac{2}{R}=\frac{1}{f}=\frac{1}{v}+\frac{1}{u}$	R	Radius of curvature of mirror.
	f	Focal length of mirror.
	v	Image distance.
	u	Object distance.
Lenses		
*I. $\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$	f	Focal length of lens.
or	v	Image distance.
II. $\frac{1}{f}=\frac{1}{v}+\frac{1}{u}$	u	Object distance.
*I. $\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$	μ	Refractive index of material composing lens.
or	R_1	Radius of curvature of surface by which light enters lens.
II. $\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$	R_2	Radius of curvature of surface by which light leaves lens.
$\frac{I}{O}=\frac{v}{u}$	I	Height of image.
	O	Height of object.
Refraction at Single Spherical Surface		
*I. $\frac{\mu}{v}-\frac{1}{u}=\frac{\mu-1}{R}$	R	Radius of curvature of surface separating two media.
or	μ	Refractive index taken in the direction of the light (i.e. air to water, glass to air, etc.)
II. $\frac{\mu}{v}+\frac{1}{u}=\frac{\mu-1}{R}$	v	Image distance.
	u	Object distance.
Telescope		
$\mathrm{M}=\frac{f_o}{f_e}$	M	Magnifying power for object at infinity.
	f_e	Focal length of eyepiece.
	f_o	Focal length of objective.

* See note on p. 21.

Compound Microscope		
$M=\frac{v}{u}\left(1+\frac{D}{f_e}\right)$	M	Magnifying power, final image at D.
	D	Least distance of distinct vision (25 cms., or 10 in.)
	v, u	Numerical values of image and object distances for objective.
	f_e	Numerical value of focal length of eyepiece.
Photometry		
$E=\frac{I \cos \theta}{d^2}$	E	Illumination of surface.
	θ	Angle of incidence of light on surface.
	d	Distance of surface from source.
	I	Luminous intensity of source.

SOUND

$V=\sqrt{\frac{T}{m}}$	V	Velocity of waves along a stretched string.
	T	Tension in string.
	m	Mass per unit length of string.
$V=n\lambda$	V	Velocity of waves in a medium.
	n	Frequency of waves.
	λ	Wavelength.
$n=\frac{1}{2L}\sqrt{\frac{T}{m}}$	n	Frequency of fundamental transverse vibration of a stretched string.
	L	Length of string.
	T	Tension in string.
	m	Mass per unit length of string.
$V_t=V_0\sqrt{\frac{273+t}{273}}$	V_t	Velocity of sound in a gas at temperature $t°$ C.
	V_0	Velocity of sound in a gas at 0° C.
Doppler Effect		
$n'=n\left(\frac{V+u}{V-v}\right)$	n	Frequency emitted by source.
	n'	Frequency received by observer.
	V	Velocity of sound in air.
	u	Velocity with which observer approaches source.
	v	Velocity with which source approaches observer.

MAGNETISM

$F=\frac{m_1 m_2}{d^2}$	F	Force which each pole exerts on the other (in air or vacuum)
	m_1, m_2	Strengths of two poles.
	d	Distance between poles.
$H=\frac{m}{d^2}$	H	Magnetic intensity or field strength at distance d from pole of strength m.
	m	
$\frac{\mathbf{V}}{\mathbf{H}}=\tan\delta$	$\mathbf{V}$	Vertical component of earth's magnetic field.
	$\mathbf{H}$	Horizontal component of earth's magnetic field.
	δ	Angle of dip.
$T=2\pi\sqrt{\frac{I}{MH}}$	T	Period of vibration.
	I	Moment of inertia of magnet about axis of vibration.
	M	Magnetic moment of magnet.
	H	Strength of field surrounding magnet.
"*End-on*" *Position* $H=\frac{2Md}{(d^2-l^2)^2}$	d	Distance of point on axis of magnet from centre of magnet
	l	Half the distance between the poles (or half the magnetic length).
"*Broadside-on*" *Position* $H=\frac{M}{(d^2+l^2)^{\frac{3}{2}}}$	d	Distance of point on the perpendicular bisector of the magnet from the centre of the magnet.
$\mathbf{B}=\mathbf{H}+4\pi\mathbf{I}$	$\mathbf{B}$	Magnetic induction.
	$\mathbf{I}$	Intensity of magnetisation.
	$\mathbf{H}$	Strength of magnetising field.
$\mu=\frac{\mathbf{B}}{\mathbf{H}}$	μ	Magnetic permeability.
$k=\frac{\mathbf{I}}{\mathbf{H}}$	k	Magnetic susceptibility.

ELECTROSTATICS

$F=\frac{Q_1Q_2}{d^2}$	Q_1, Q_2	Two point charges
	F	Force which each exerts on the other (in air).
	d	Distance between charges.
$V=\frac{Q}{d}$	V	Potential at a point distant d from a charge Q.
$C=\frac{Q}{V}$	C	Capacity of a conductor at potential V and with charge Q.
$C=\frac{Ak}{4\pi d}$	C	Capacity of a parallel plate condenser.
	A	Area of each plate.
	d	Distance between plates.
	k	Specific inductive capacity (dielectric constant) of medium between plates.
$C=\frac{kRr}{R-r}$	C	Capacity of a spherical condenser.
	r	Radius of inner sphere.
	R	Radius of outer spherical shell.
Condensers in Series $\frac{1}{C}=\frac{1}{c_1}+\frac{1}{c_2}+\frac{1}{c_3}+\ldots$	c	Individual capacities.
	C	Combined capacity in series.
Condensers in Parallel $C=c_1+c_2+c_3+\ldots$	C	Combined capacity in parallel.
$W=\frac{1}{2}QV$ $=\frac{1}{2}CV^2$ $=\frac{1}{2}\frac{Q^2}{C}$	W	Energy of charge of condenser.

ELECTRICITY

Ohm's Law		
$I=\dfrac{V}{R}$	I	Current.
	V	Potential difference.
or		
$I=\dfrac{E}{R}$	E	Electromotive force.
	R	Resistance.
Circular Coil		
$H=\dfrac{2\pi ni}{r}$	H	Axial field at centre of circular coil.
	n	Number of turns in coil.
	r	Radius of coil.
	i	Current in absolute electromagnetic units.
Tangent Galvanometer		
$i=\dfrac{r\mathbf{H}}{2\pi n}\tan\theta$	$\mathbf{H}$	Earth's horizontal magnetic field strength.
or	n	Number of turns in coil.
$I=\dfrac{5r\mathbf{H}}{\pi n}\tan\theta$	r	Radius of coil.
	θ	Angle of deflection.
	i	Current in absolute units.
	I	Current in amperes.
Resistances in Series		
$R=r_1+r_2+r_3+\ \ldots$	r	Individual resistances.
	R	Combined resistance in series.
Resistances in Parallel		
$\dfrac{1}{R}=\dfrac{1}{r_1}+\dfrac{1}{r_2}+\dfrac{1}{r_3}+\ \ldots$	R	Combined resistance in parallel.
$R=\dfrac{\sigma L}{a}$	σ	Specific resistance (resistivity) of material.
	L	Length of wire.
	a	Cross-sectional area of wire.
$W=IVt$ $=I^2Rt$ $=\dfrac{V^2t}{R}$	W	Energy dissipated in a conductor in time t.
	t	Time.
$W=JH$	H	Heat equivalent of energy W.
	J	Mechanical equivalent of heat $=4{\cdot}18\times10^7$ ergs/cal. $=4{\cdot}18$ joules/cal.

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SIGN CONVENTIONS

OWING to the lack of uniformity in the sign conventions recommended by teachers of optics, it has been thought advisable, in lens problems, to give two solutions, side by side, one for each of the two most popular conventions. These are as follows:

(I) All distances to be measured from the surface of the lens or mirror. The direction of the incident light to be taken as negative.

* (II) Distances of real objects and images to be positive; of virtual objects and images to be negative.

Concave reflecting or convex refracting surfaces to have positive radii of curvature; convex reflecting or concave refracting surfaces, negative radii of curvature.

In the above table, the two versions of the lens formulæ correspond to these two conventions and are appropriately distinguished as (I) or (II).

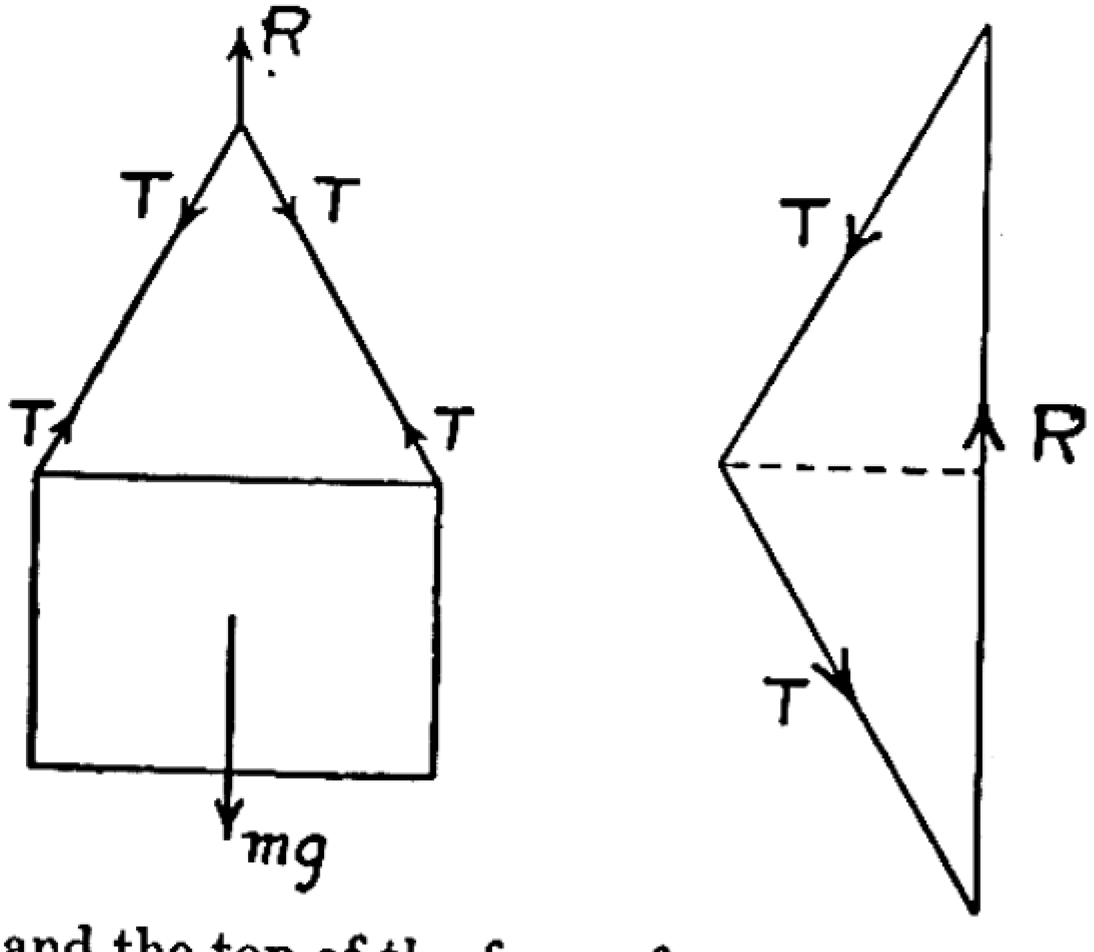
* The author finds some difficulty in the use of (II) for the more complex problems. Inconsistencies seem to be unavoidable unless the statement of the convention is greatly extended to include additional conditions as to the signs of radii of curvature. (I) appears preferable on account of the ease with which it can be applied rigidly in all cases.

GENERAL PHYSICS

MECHANICS, HYDROSTATICS, SURFACE TENSION, ELASTICITY

1.

A RECTANGULAR picture frame 18 in. wide is suspended from a nail by a cord 3 ft. long whose two ends are attached to the two top corners of the frame. Calculate the tension in the cord if the mass of the frame is 5 lb.



The cord and the top of the frame form an equilateral triangle, so that each portion of the cord makes an angle of 30° with the vertical. The forces on the frame are therefore its weight mg vertically down and the two tensions T in the cord up at an angle of 30° to the vertical.

Resolving the forces vertically:

$$mg = 2T \cos 30° \text{ for equilibrium}$$

$$\therefore \quad 5 \times 32 = 2T \times \frac{\sqrt{3}}{2}$$

mg = 2T cos 30°
for equilibrium

$$T = \frac{5 \times 32}{\sqrt{3}} \text{ poundals}$$

$$= \underline{92{\cdot}4 \text{ poundals}} \text{ or } \underline{2{\cdot}89 \text{ lb. wt.}}$$

4 . poundals

The problem may also be solved by the triangle of forces applied to the three forces acting on the nail. These are the reaction of the wall R vertically up and the two tensions T. The triangle of forces drawn with its sides parallel to the three forces in equilibrium, is as shown above, from which it may be seen that

$$R=2T\cos 30°$$

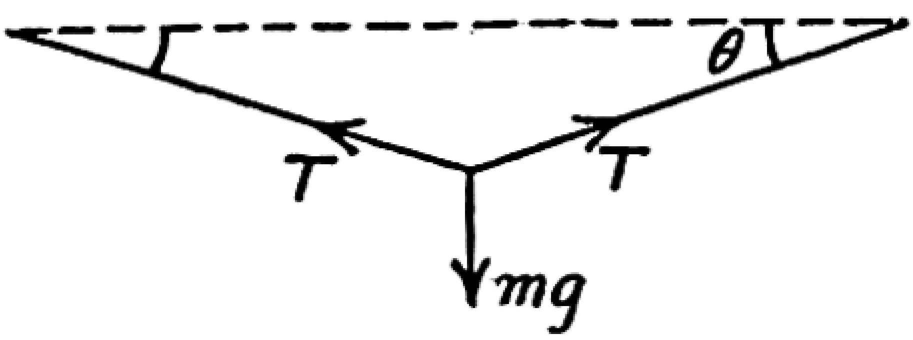
But since the nail is supporting the frame, the reaction of the wall R must equal the weight of the frame.

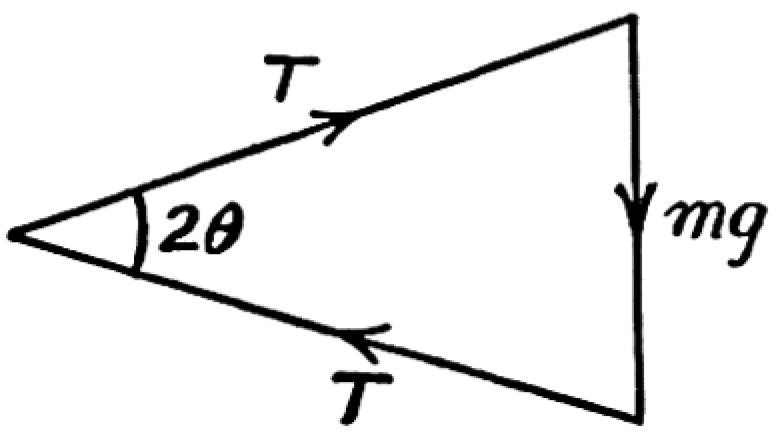
$$\therefore \qquad 5\times 32=2T\cos 30°$$

which is the same equation as was obtained by the first method.

2.

A tight-rope walker of mass 10 stone, standing at the centre of a rope stretched between two points at the same level and 30 ft. apart, produces a sag of 2 ft. Find the tension in the rope.





Triangle of forces

For equilibrium of the three forces acting at the centre of the rope:

$$mg=2T\sin\theta$$

(vertical components of T).

Since $\tan\theta=\frac{2}{15}=0{\cdot}1333$

$$\theta=7°\ 36'$$

and $\sin\theta=0{\cdot}1323$

$$\therefore \qquad T=\frac{140\times 32}{2\times 0{\cdot}1323}$$

$$=16{,}930 \text{ poundals}$$

$$=\underline{529 \text{ lb. wt.}}$$

Note that for small values of θ, for which $\sin\theta$ and $\tan\theta$ may be taken as equal, the product of the sag and the tension is constant.

3.

Find the greatest slope up which a man could push a railway truck weighing 2 tons, if the maximum force which he can exert is 112 lb. wt.

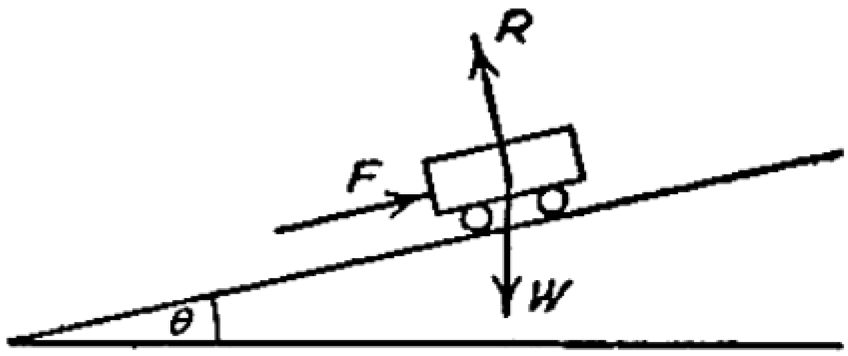
In the diagram the three forces acting on the truck are shown:

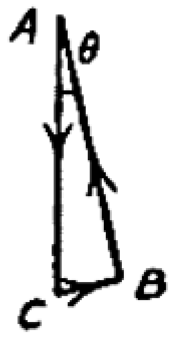
W, its weight acting vertically,

R, the normal reaction of the rails,

F, the force exerted by the man parallel to the rails.

For equilibrium, the relative magnitudes of the forces may be represented by the three sides of a triangle drawn parallel to the lines of action of the three forces.





ABC is such a "triangle of forces" in which

BA is parallel to R.
AC is parallel to W.
CB is parallel to F.

The angle CAB is equal to θ, the slope of the rails.
So we may write:

$$\sin\theta = \frac{\mathrm{CB}}{\mathrm{AC}} = \frac{F}{W}$$

$$= \frac{112}{2 \times 2240} = \frac{1}{40}$$

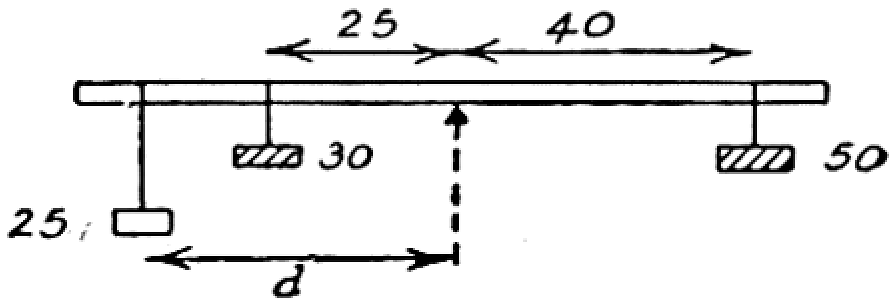
$$\theta = 1^\circ\ 30'$$

If the slope is less than this, the forces will no longer be in equilibrium and the truck will move up.

The greatest slope up which the truck can be pushed is one inclined at $1^\circ\ 30'$ to the horizontal—a slope of 1 in 40.

4.

A metre rule is pivoted at its mid-point, and masses of 30 and 50 gm. are hung at the 25-cm. and 90-cm. divisions respectively. Where would it be necessary to hang a mass of 25 gm. in order that the rule should balance horizontally?



Let the required position of the 25-gm. mass be d cm. from the pivot.

Then, for equilibrium, taking moments about the pivot:

$$(30\times 25)+(25\times d)=(50\times 40)$$

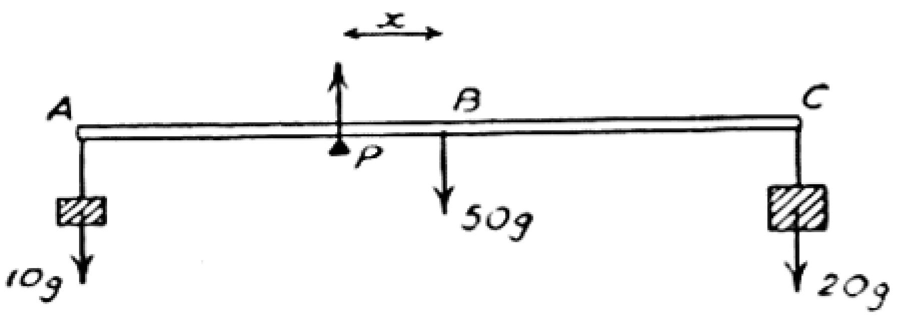
$$d=\frac{1}{25}(2000-750)$$

$$=50 \text{ cm.}$$

Therefore the 25-gm. mass must be hung at the extreme end of the rule on the same side of the pivot as the 30-gm. mass.

5.

A uniform lath 1 metre long of mass 50 gm. has a mass of 10 gm. attached to one end, and a mass of 20 gm. attached to the other. Find the position of the point on the lath about which it would balance horizontally.



Suppose the required point of support is at P, at a distance x to the left of the centre of the lath. Then there are four forces acting on the lath:

Its weight acting through the mid-point B.
10 gm. wt. acting through the end A.
20 gm. wt. acting through the end C.
The reaction of the support acting up through P.

For equilibrium, taking moments about P:

$$10\times AP=(50\times PB)+(20\times PC)$$

$$\therefore\quad 10(50-x)=50x+20(50+x)$$

$$-500=80x$$

$$x=\frac{-50}{8}$$

$$=-6{\cdot}25 \text{ cm.}$$

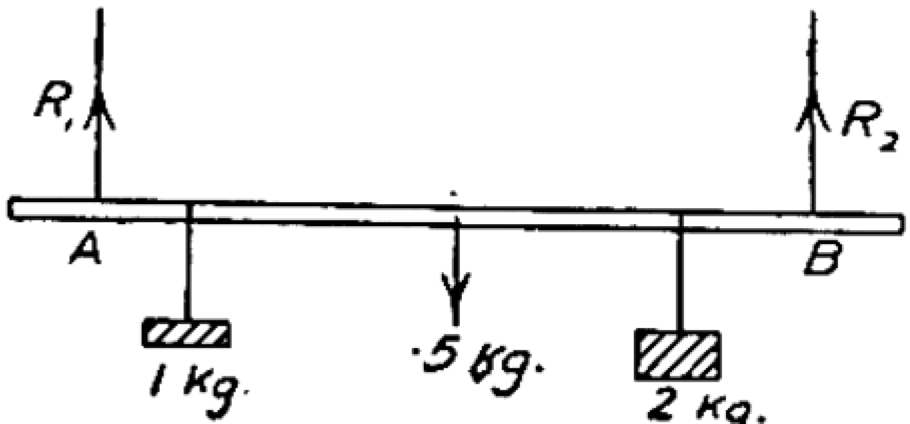
The negative sign indicates that P is to the *right* of B.

So the point of balance is 6·25 cm. from the centre of the lath on the side of the larger mass.

6.

A uniform rod 1 metre long, of mass 500 gm., is suspended in a horizontal position by two vertical spring-balances at points 10 cm. from each end. Masses of 1 and 2 kgm. are hung on the rod at points respectively 20 cm. from one end and 25 cm. from the other. What are the readings of the spring-balances?

Let the balance readings be R_1 at A (the 1-kgm. side) and R_2 at B (the 2-kgm. side). Then, taking moments about A:



$$(1\times10)+(0{\cdot}5\times40)+(2\times65)=(R_2\times80)$$

$$R_2=\frac{1}{80}(10+20+130)$$

$$=\underline{2\text{ kgm. wt.}}$$

Taking moments about B:

$$(2\times15)+(0{\cdot}5\times40)+(1\times70)=(R_1\times80)$$

$$R_1=\frac{120}{80}$$

$$=\underline{1{\cdot}5\text{ kgm. wt.}}$$

As a check on the answer, note that the sum of the spring-balance readings equals the sum of the two suspended masses and the mass of the rod.

7.

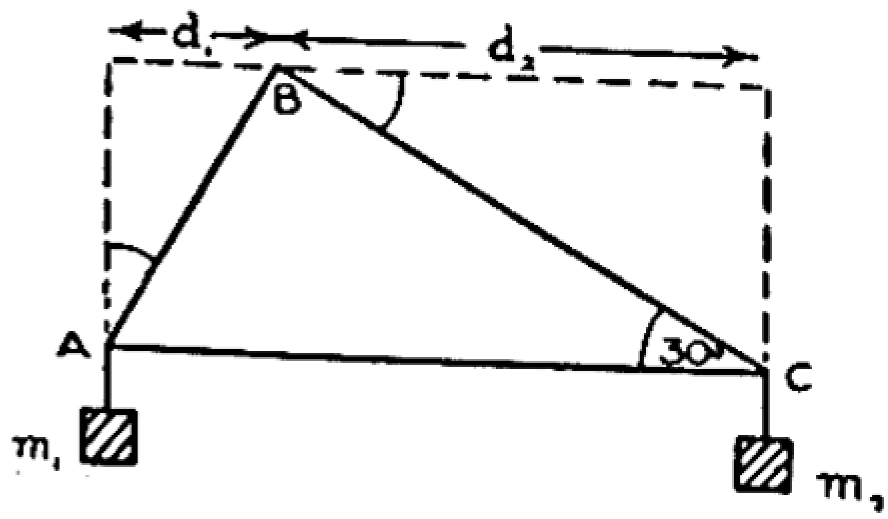
A 30°, 60° triangular set-square of negligible mass is suspended freely from the right-angled corner, and weights are hung at the other two corners. What is the ratio of the masses of the weights if the hypotenuse of the set-square sets horizontally?

Referring to the diagram, we may take moments about B, and for equilibrium if AC is horizontal:

$$m_1d_1=m_2d_2$$

$$m_1\text{ AB}\sin 30°=m_2\text{ BC}\cos 30°$$

$$\frac{m_1}{m_2}=\frac{\text{BC}\cos 30°}{\text{AB}\sin 30°}$$

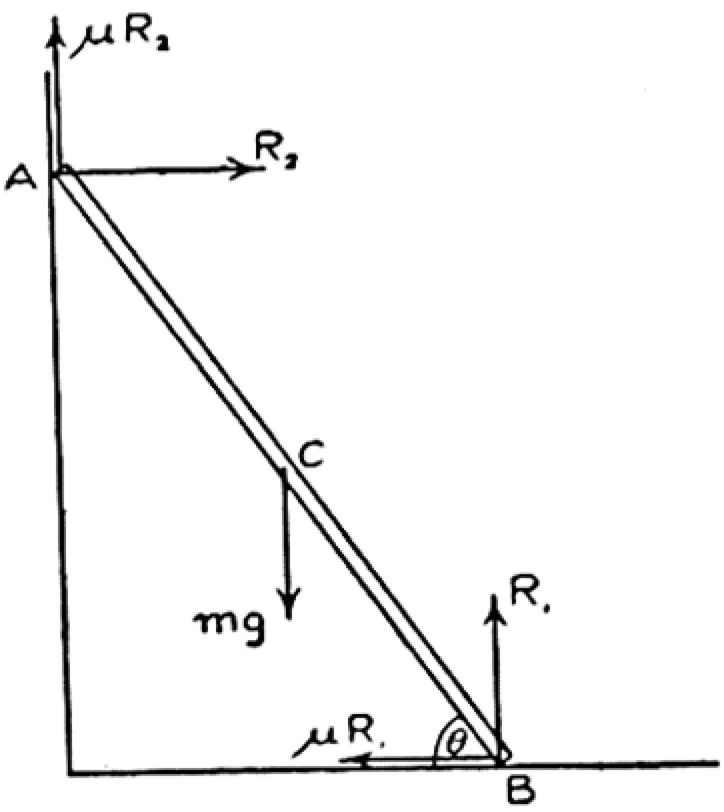


$$=\frac{BC}{AB}\cot 30°$$
$$=\cot 30°\times\cot 30°$$
$$=\cot^2 30°$$
$$=(\sqrt{3})^2$$
$$=3$$

The ratio of the masses is 3 : 1, the larger mass being at the 60° corner.

8.

A uniform plank leans against a vertical wall, the coefficient of friction at each end of the plank being 0·25. What is the minimum angle at which the plank may be inclined to the horizontal without slipping?



Let θ be the inclination of the plank as it is about to slip.

The forces acting on the plank when just on the point of slipping will be as in the diagram, R_1 and R_2 being the reactions of the ground at B and the wall at A, μR_1 and μR_2 the limiting frictional forces resisting slipping, and mg the weight of the plank acting through its mid-point C.

For equilibrium, resolving all the forces acting on the plank:

$$\left.\begin{array}{lr}(a)\ \text{horizontally:} & R_2=\mu R_1\\ (b)\ \text{vertically:} & mg=\mu R_2+R_1\end{array}\right\} mg=\mu^2 R_1+R_1$$

Taking moments about A:

$$2R_1\cos\theta=mg\cos\theta+2\mu R_1\sin\theta$$

Substituting for mg:

$$2R_1\cos\theta=R_1(\mu^2+1)\cos\theta+2\mu R_1\sin\theta$$
$$\cos\theta[2-(\mu^2+1)]=2\mu\sin\theta$$
$$\therefore\qquad \tan\theta=\frac{2-(\mu^2+1)}{2\mu}$$

$$=\frac{1-\mu^2}{2\mu}$$

$$=\frac{1-0{\cdot}0625}{0{\cdot}5}$$

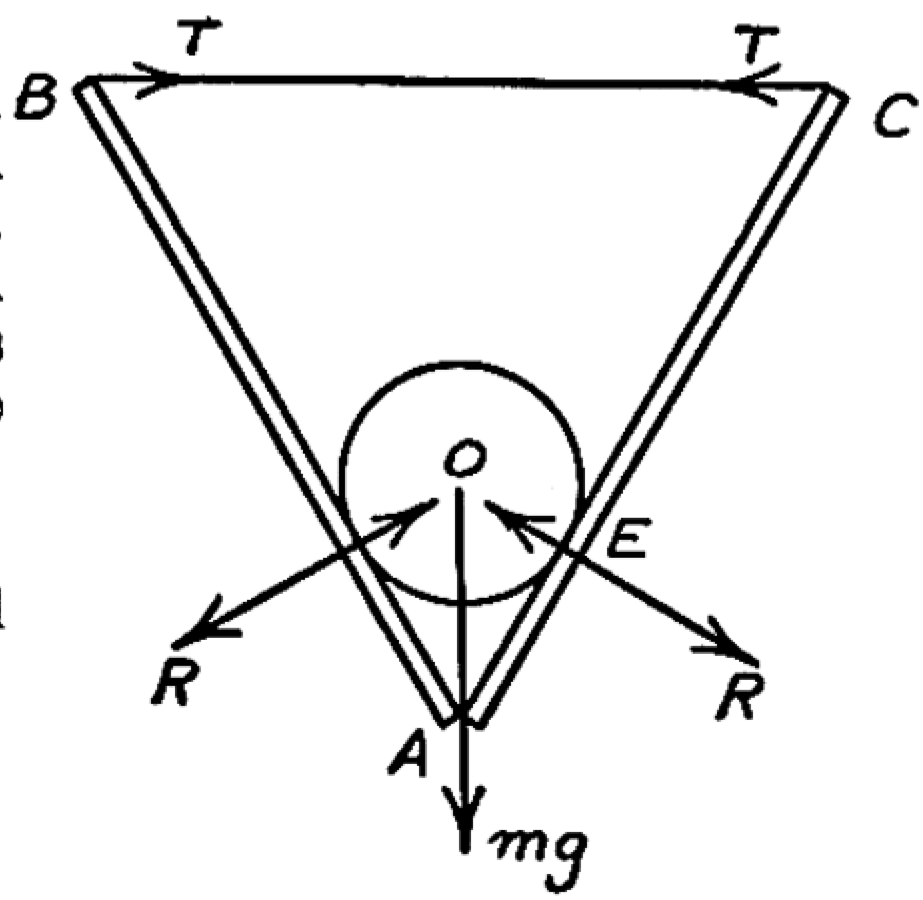
$$=1{\cdot}875$$

$$\theta=61°\ 56'$$

The minimum angle between the plank and the horizontal is 61° 56′.

9.

A pair of flat boards 1 ft. square are hinged together along one edge, and the opposite edges are joined by a string 1 ft. long. The boards are held in the position of a V-shaped trough with sides equally inclined to the vertical and a metal sphere of mass 2 lb. and radius 2 in. is placed within it. What is the tension in the string, neglecting the masses of the boards?



Referring to the diagram, R is the normal force exerted by the sphere on each board. For equilibrium, the vertical components of these forces must together equal the weight of the sphere.

$\therefore\ 2R\cos 60°=2\times 32$

since ABC is an equilateral triangle.

$$R=64 \text{ poundals}$$

The distance of the point of contact E from the hinge A is:

$$\text{EA}=\text{OE}\tan 60°$$

$$=2\times\sqrt{3}\text{ in.}$$

$$=3{\cdot}464\text{ in.}$$

Taking moments now about A:

$$R \times \mathrm{AE} = T \cos 30° \times \mathrm{AC}$$

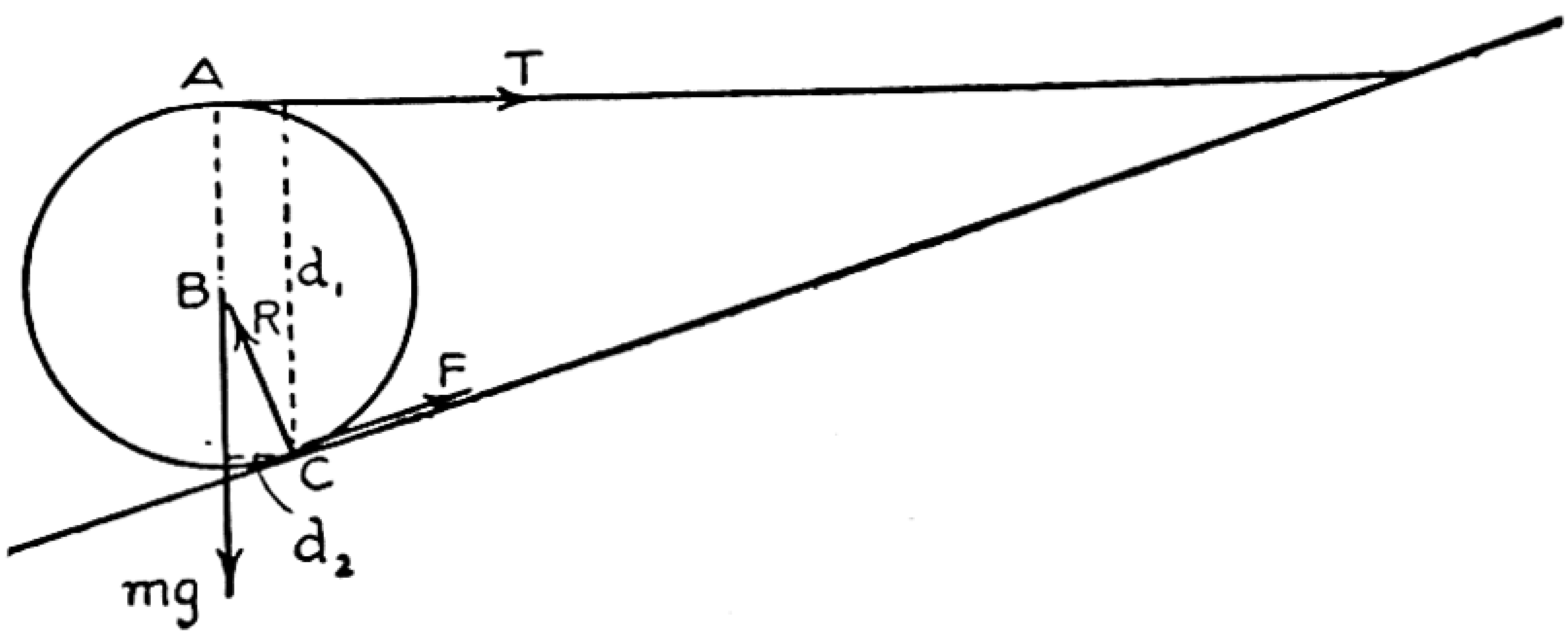
$$64 \times 2 \times \sqrt{3} = T \times \frac{\sqrt{3}}{2} \times 12$$

$$T = \frac{64 \times 2}{6} \text{ poundals}$$

$$= \frac{2}{3} \text{ lb. wt.}$$

10.

A cylinder of diameter 50 cm. and mass 4 kgm. is prevented from rolling down a rough slope inclined at 20° to the horizontal by a cord, one end of which is attached to the surface of the cylinder and the other end to a point in the plane, the cord being horizontal and tangential to the cylinder. Find the tension in the cord.



The forces acting on the cylinder are:

Its weight acting vertically down through B, the centre of gravity.

The horizontal tension in the cord acting through A.

The normal reaction of the plane at C.

A frictional force acting at C parallel to the plane.

These last two forces do not enter into the calculation if we take moments about C:

For equilibrium, $Td_1 = mgd_2$

$$d_2 = \text{BC} \sin 20°$$
$$= 25 \times 0{\cdot}342$$
$$= 8{\cdot}55 \text{ cm.}$$
$$d_1 = \text{AB} + \text{BC} \cos 20°$$
$$= 25 + (25 \times 0{\cdot}940)$$
$$= 48{\cdot}5 \text{ cm.}$$

$\therefore$ $$T \times 48{\cdot}5 = 4000 \times 981 \times 8{\cdot}55$$
$$T = \frac{4000 \times 981 \times 8{\cdot}55}{48{\cdot}5} \text{ dynes}$$
$$= \frac{4000 \times 8{\cdot}55}{48{\cdot}5} \text{ gm. wt.}$$
$$= \underline{705 \text{ gm. wt.}}$$

11.

A pebble is dropped from rest from the top of a cliff 500 ft. high. How long does it take to reach the bottom? What is its velocity just before striking the ground?

In the equation: $$s = ut + \frac{1}{2}at^2$$

we are given $$\begin{cases} s = 500 \text{ ft.} \\ u = 0 \end{cases}$$

and we know $$a = g = 32 \text{ ft./sec.}^2$$

$\therefore$ $$500 = 0 + \left(\frac{1}{2} \times 32 \times t^2\right)$$
$$t = \sqrt{\frac{500}{16}} \text{ sec.}$$
$$= \frac{\sqrt{500}}{4}$$
$$= \frac{10\sqrt{5}}{4}$$
$$= \frac{10 \times 2{\cdot}24}{4}$$
$$= \underline{5{\cdot}6 \text{ sec.}}$$

The average velocity is therefore $\frac{500}{5\cdot6}$ ft./sec., and since the initial velocity was zero, the final velocity

$$=\frac{2\times500}{5\cdot6}$$
$$=\underline{179 \text{ ft./sec.}}$$

Or we may use the equation:

$$v=u+at$$

obtaining

$$v=0+(32\times5\cdot6)$$
$$=\underline{179 \text{ ft./sec.}}$$

The pebble takes 5·6 sec. to fall 500 ft. to the bottom of the cliff, its final velocity being 179 ft. per sec.

12.

A train, starting from rest, attains a speed of 60 miles per hour in 11 mins. with uniform acceleration. How far does it travel during (*a*) the first minute, (*b*) the second minute, (*c*) the eleventh minute?

The equations required for this problem are:

$$v=u+at$$
$$s=ut+\frac{1}{2}at^2$$

We are given

$$v=60 \text{ m.p.h.}=\frac{60\times5280}{3600} \text{ ft./sec.}$$
$$=88 \text{ ft./sec.}$$
$$u=0$$

as the train starts from rest

$$t=11 \text{ min.}=660 \text{ sec.}$$

From the first equation:

$$88=0+660a$$
$$a=\frac{2}{15} \text{ ft./sec.}^2$$

(*a*) In the *first minute* $t=60$ sec.

$$\therefore \quad s_1=0+\left(\frac{1}{2}\times\frac{2}{15}\times60^2\right)$$
$$=\underline{240 \text{ ft.}}$$

(*b*) In the first two minutes

$$t=120 \text{ sec.}$$

$$s_2=\left(\frac{1}{2}\times\frac{2}{15}\times 120^2\right)$$

$$=960 \text{ ft.}$$

∴ Distance travelled in the *second minute*

$$=960-240$$

$$=\underline{720 \text{ ft.}}$$

(*c*) In 11 mins. $\quad s_{11}=\left(\frac{1}{2}\times\frac{2}{15}\times 660^2\right)$

$$=29{,}040 \text{ ft.}$$

In 10 mins. $\quad s_{10}=\left(\frac{1}{2}\times\frac{2}{15}\times 600^2\right)$

$$=24{,}000 \text{ ft.}$$

∴ Distance travelled in the *eleventh minute*

$$=29{,}040-24{,}000$$

$$=\underline{5040 \text{ ft.}}$$

Alternative Method

The problem may also be worked for (*b*) and (*c*) by calculating the velocity at the beginning of each of the specified periods and substituting this for u in the equation:

$$s=ut+\frac{1}{2}at^2$$

Thus:

(*b*) From $v=u+at$, the velocity at the beginning of the second minute, i.e. after 1 min., is

$$0+\left(\frac{2}{15}\times 60\right)=8 \text{ ft./sec.}$$

And from $s=ut+\frac{1}{2}at^2$,

$$s=(8\times 60)-\left(\frac{1}{2}\times\frac{2}{15}\times 60^2\right) \text{during the second minute}$$

$$=480+240$$

$$=\underline{720 \text{ ft.}}$$

(*c*) The velocity at the beginning of the eleventh minute, i.e. after 10 min., is

$$0+\left(\frac{2}{15}\times 10\times 60\right)=80 \text{ ft./sec.}$$

$$\therefore \qquad s=(80\times 60)+\left(\frac{1}{2}\times\frac{2}{15}\times 60^2\right) \text{during the eleventh minute}$$

$$=4800+240$$
$$=\underline{5040 \text{ ft.}}$$

13.

With what velocity must a bullet be fired vertically in order to reach a height of 2 miles? What interval will elapse after firing before the bullet returns to earth?

In the three equations of motion

(1) $$s=ut+\frac{1}{2}at^2$$

(2) $$v=u+at$$

(3) $$v^2=u^2+2as$$

we are given, for the upward journey:

$$s=2\times 5280 \text{ ft.}$$
$$v=0$$

(final velocity momentarily zero at the top)

$$a=-g=-32 \text{ ft./sec.}^2$$

and we require u and t. From (3):

$$0=u^2-(2\times 32\times 2\times 5280)$$
$$\therefore \qquad u=\sqrt{64\times 10560} \text{ ft./sec.}$$
$$=8\times 102{\cdot}7$$
$$=822 \text{ ft./sec.}$$

The bullet must be fired with a velocity of 822 ft./sec.

From (2): $$0=822-32t$$
$$\therefore \qquad t=25{\cdot}7 \text{ sec.}$$

This is the time for the upward journey; that for the downward journey will be the same; therefore:

51·4 sec. will elapse between the firing of the gun and the return of the bullet to earth.

14.

A car of mass 1 ton, travelling at 15 miles per hour, can be stopped by its brakes in 30 ft. What is the average retarding force of the brakes?

$$15 \text{ m.p.h.} = \frac{88}{4} \text{ ft. per sec.}$$

Substituting in the equation

$$v^2 = u^2 + 2as$$

we have
$$0 = 22^2 + (2a \times 30)$$

$$\therefore \quad a = -\frac{484}{60}$$
$$= -8{\cdot}07 \text{ ft./sec.}^2$$

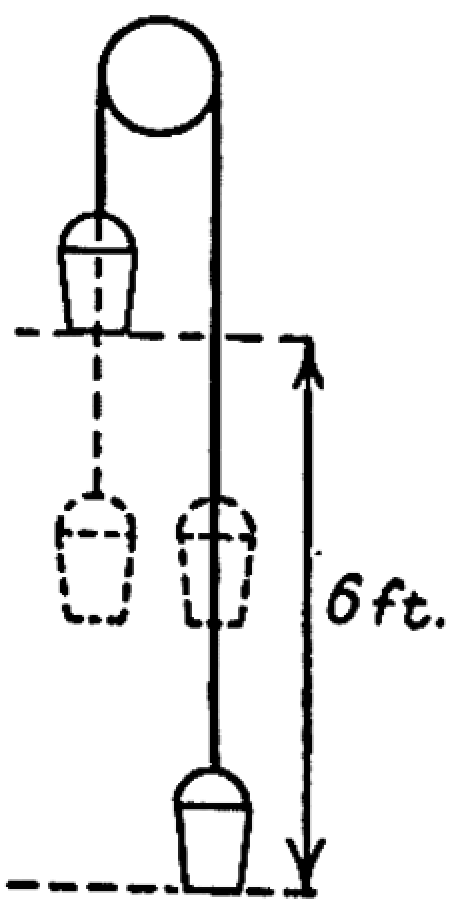
the negative sign indicating a retardation.

The force required to produce this retardation in a mass of 2240 lb.

$$= 2240 \times 8{\cdot}07 \text{ poundals}$$
$$= \frac{2240 \times 8{\cdot}07}{32} \text{ lb. wt.}$$
$$= \underline{565 \text{ lb. wt.}}$$

15.

Two similar buckets, each of mass 3 lb., are suspended from the two ends of a long cord of negligible mass passing over a frictionless pulley and are at rest at the same level. A mass of 2 oz. is now placed gently in one of the buckets. How long will it be before the vertical distance between the buckets is 6 ft.?



Each bucket has moved through a distance of 3 ft. The accelerating force is the weight of 2 oz.

$$\text{or } \left(\frac{2}{16} \times 32\right) \text{ poundals.}$$

The total mass being accelerated is $(2 \times 3) + \frac{1}{8}$ lb.

The acceleration a is equal to the force divided by the mass (from $F = ma$).

$$\therefore \quad a = \left(\frac{2 \times 32}{16} \times \frac{1}{6\frac{1}{8}}\right) \text{ ft./sec.}^2$$
$$= \frac{32}{49} \text{ ft./sec.}^2$$

We require the time taken by each bucket to travel 3 ft.

Substituting the known values of s, u and a in the equation

$$s=ut+\frac{1}{2}at^2$$

we have

$$3=0+\left(\frac{1}{2}\times\frac{32}{49}\times t^2\right)$$

$$t^2=\frac{3\times 49}{16}$$

$$t=\frac{7\sqrt{3}}{4}=3{\cdot}03 \text{ sec.}$$

The time taken by the buckets to separate by 6 ft. is 3·03 sec.

16.

A bullet of mass 20 gm. is fired from a rifle having a barrel 50 cm. long and 8 mm. in diameter. If the average excess pressure in the barrel is 5000 atmospheres, calculate the muzzle velocity of the bullet, neglecting the effects of friction and of the rifling of the barrel.

Density of mercury=13·6 gm./c.c.

In order to apply the equations of motion, it is necessary to find the accelerating force in dynes acting on the bullet.

A pressure of 1 atmosphere

=the pressure due to 76 cm. of mercury

$=(76\times 13{\cdot}6\times 981)$ dynes per sq. cm.

Average pressure in the barrel

$=(5000\times 76\times 13{\cdot}6\times 981)$ dynes per sq. cm.

$=5{\cdot}07\times 10^9$ dynes per sq. cm.

Average force on bullet of diameter 0·8 cm.

$=5{\cdot}07\times 10^9\times \pi\times 0{\cdot}4^2$ dynes

$=2{\cdot}55\times 10^9$ dynes

From the equation, $F=ma$, we now have:

$$2{\cdot}55\times 10^9=20a$$

$$\therefore \quad a=1{\cdot}275\times 10^8 \text{ cm./sec.}^2$$

and from the equation:

$$v^2=u^2+2as$$

in which $u=0$ and $s=50$ cm., we have

$$v^2=0+(2\times 50\times 1{\cdot}275\times 10^8)$$

$$\therefore \quad v=\sqrt{1{\cdot}275\times 10^5} \text{ cm. per sec.}$$

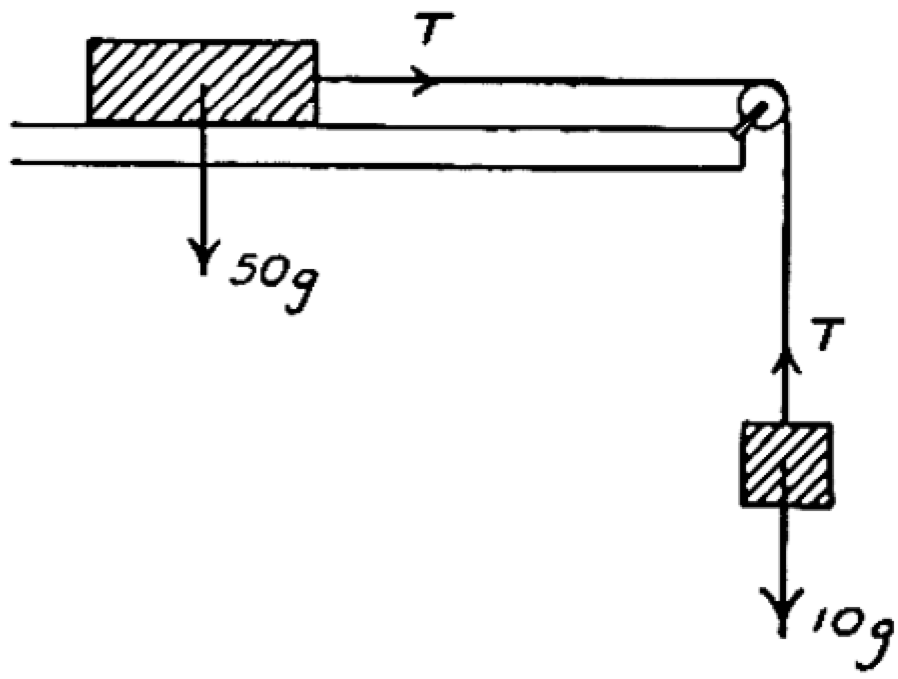
$=1{\cdot}129\times 10^3$ metres per sec.

$=$1·129 km. per sec.

17.

A wooden block of mass 50 gm. resting on a smooth table is connected by a cord passing over a frictionless pulley at the edge of the table to a mass of 10 gm. hanging freely, the two portions of the cord being respectively horizontal and vertical. With what acceleration will the block slide along the table? What is the tension in the cord?

The force causing motion of the two masses is the weight of the suspended mass, $10g$ dynes. The total mass being accelerated is $(50+10)$ gm.



Therefore, applying the equation

$$F=ma$$

we have

$$10g=(50+10)a$$

$$a=\frac{10g}{60}$$

$$=\frac{981}{6}\text{ cm./sec.}^2$$

$$=\underline{163{\cdot}5\text{ cm./sec.}^2}$$

Considering the motion of the block alone:
the accelerating force is the tension in the cord, T.

$$\therefore \qquad T=50\times 163{\cdot}5$$

$$=\underline{8175\text{ dynes}}$$

Or, considering the motion of the hanging mass alone:
the resultant vertical accelerating force is $(10g-T)$ dynes.

$$\therefore \qquad 10g-T=10\times 163{\cdot}5$$

$$T=9810-1635$$

$$=\underline{8175\text{ dynes}}$$

the same result as was obtained above.

Notice that the tension in the cord would be $10g=9810$ dynes if the masses were prevented from moving.

18.

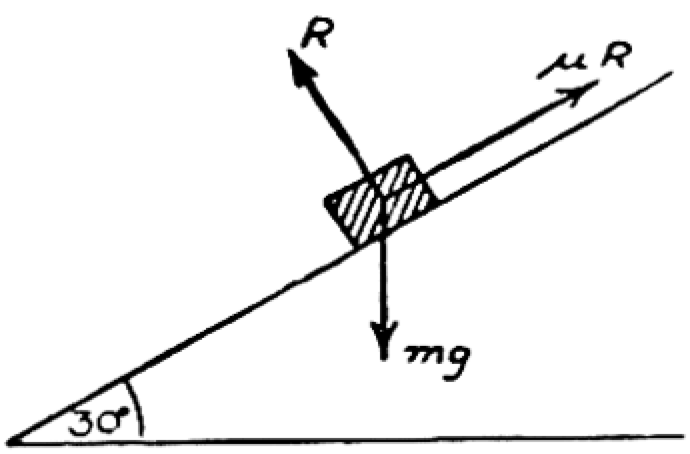
How long will it take for a wooden block to slide from rest a distance of 3 metres down a slope inclined at 30° to the horizontal, if the coefficient of friction between the block and the surface of the slope is 0·20?

The three forces acting on the block are not in equilibrium, but have a resultant component parallel to the plane which accelerates the block down the plane.

This accelerating force is equal to

$$mg \sin 30° - \mu R$$

Since there is no component of the motion perpendicular to the plane, the resultant force in this direction must be zero.



$\therefore \quad R = mg \cos 30°$

and $\quad \mu R = 0{\cdot}2mg \cos 30°$

So the accelerating force is:

$$mg \sin 30° - 0{\cdot}2mg \cos 30°$$

$$= mg\left(0{\cdot}5 - \frac{0{\cdot}2\sqrt{3}}{2}\right)$$

$$= 0{\cdot}327mg$$

This must equal the product of the mass and the acceleration

$\therefore \quad 0{\cdot}327mg = ma$

$$a = 0{\cdot}327 \times 981 \text{ cm./sec.}^2$$

Now in the equation, $s = ut + \frac{1}{2}at^2$

we have $\quad s = 300$ cm.

$u = 0$ since the block starts from rest

and $\quad a = 0{\cdot}327 \times 981$ cm. per sec.^{2}

$$\therefore \quad 300 = \frac{1}{2} \times 0{\cdot}327 \times 981 \times t^2$$

$$t = \sqrt{\frac{2 \times 300}{0{\cdot}327 \times 981}}$$

$$= 1{\cdot}37 \text{ sec.}$$

It will take 1·37 sec. for the block to slide a distance of 3 metres down the slope.

19.

A lead sphere of mass 20 gm. is whirled round in a horizontal circular path on the end of a string 1 metre long, the other end of which is stationary. What is the tension in the string when the sphere is making 4 revolutions per second?

The radial acceleration for a body following a circular path of radius R with velocity v is:

$$a=\frac{v^2}{R}$$

The centrifugal force on mass m is:

$$ma=\frac{mv^2}{R}$$

which is balanced by the tension in the string.

$$\therefore \qquad \text{Tension}=\frac{20\times(2\pi\times100\times4)^2}{100}\text{ dynes}$$
$$=128{,}000\pi^2 \text{ dynes}$$
$$=1{\cdot}263\times10^6 \text{ dynes}$$
$$=\frac{1{\cdot}263\times10^3}{981}\text{ kgm. wt.}$$
$$=\underline{1{\cdot}288 \text{ kgm. wt.}}$$

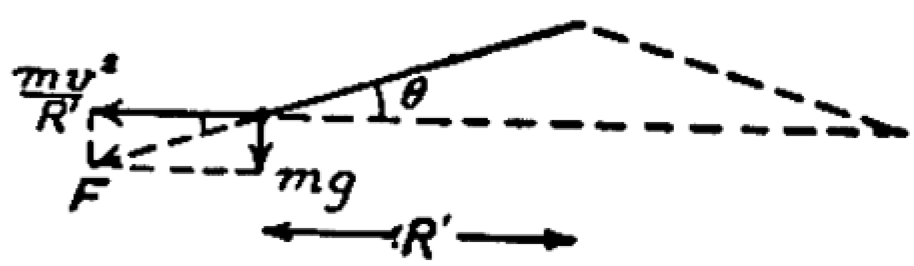
It has been assumed in the above solution that the string is horizontal. Actually this would not be so. The string would be in the direction of the resultant (F) of the vertical weight (mg) of the sphere, and the horizontal centrifugal force $\left(\frac{mv^2}{R'}\right)$

$$\therefore \qquad \tan\theta=\frac{mg}{\frac{mv^2}{R'}}=\frac{gR'}{v^2}$$

$$=\frac{981\times100}{(2\pi\times100\times4)^2}\text{ approx.}$$

$$=\frac{981}{6400\pi^2}$$

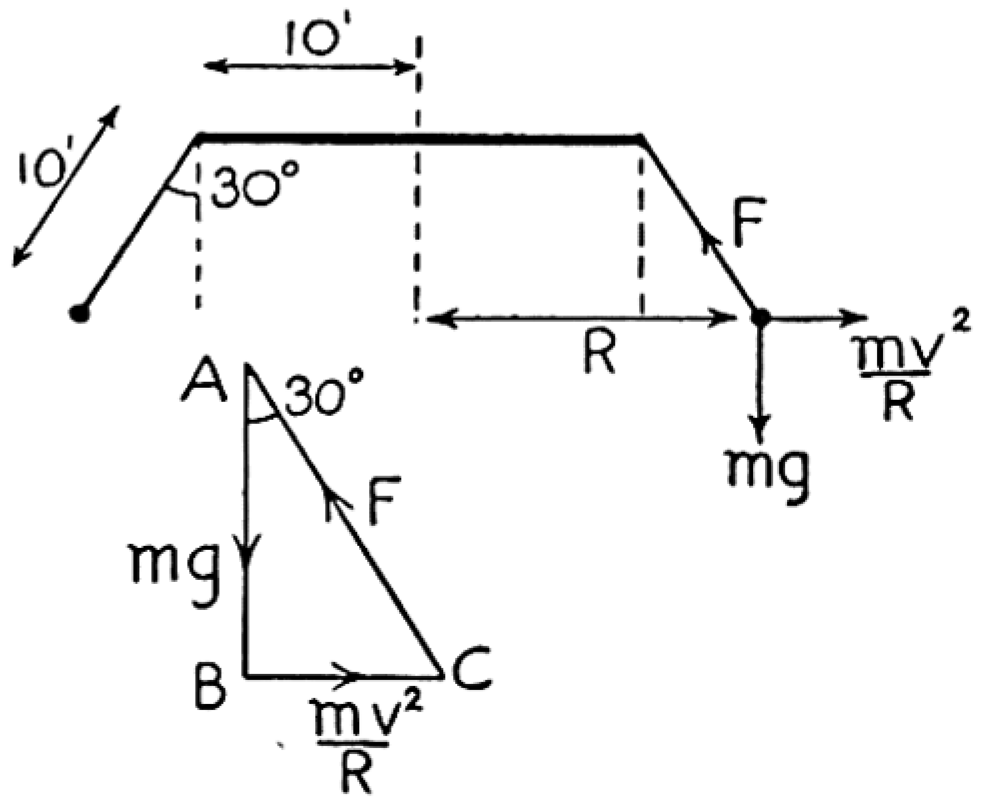
$$=\frac{1}{64}\text{ roughly}$$



θ would therefore be less than 1°, its cosine being 0·9999. R' would differ from 1 metre, and the tension from $\frac{mv^2}{R}$ by about 1 part in 10,000.

20.

A horizontal disc 20 ft. in diameter has a number of ropes, each 10 ft. long, attached to its rim, each rope carrying a weight. When the disc rotates at a uniform rate, the ropes are seen to be inclined to the vertical at an angle of 30°. What is the time for one revolution of the disc?



Referring to the diagram, it is seen that the three forces acting on each weight are: the weight mg vertically down, the tension F in the rope and the centrifugal force $\frac{mv^2}{R}$ acting horizontally.

Since these are in equilibrium, and can therefore be represented by the triangle of forces ABC, it is obvious that

$$\tan 30° = \frac{\frac{mv^2}{R}}{mg} = \frac{v^2}{Rg}$$

Now if T is the required period of revolution of the disc, during which time the weight travels once round a circle of radius R,

$$v = \frac{2\pi R}{T}$$

and, substituting for v in the previous equation,

$$\tan 30° = \left(\frac{2\pi R}{T}\right)^2 \cdot \frac{1}{Rg}$$

$$= \frac{4\pi^2 R}{T^2 g}$$

from which

$$T^2 = \frac{4\pi^2 R}{g \tan 30°}$$

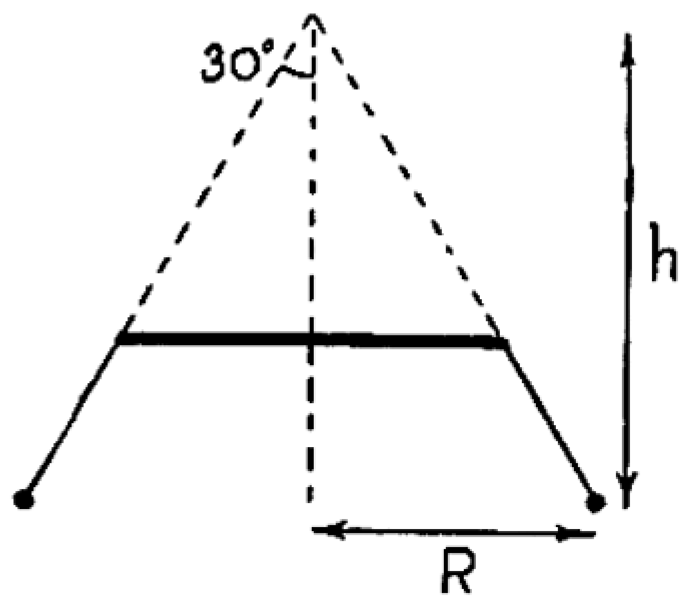
$$R = 10 + 10 \sin 30° = 15 \text{ ft.}$$

$$\tan 30° = \frac{1}{\sqrt{3}}$$

$$\therefore \qquad T^2 = \frac{4\pi^2 \times 15 \times \sqrt{3}}{32}$$

$$T = \underline{5{\cdot}66 \text{ sec.}}$$

The problem may be solved more quickly by considering the system as part of a conical pendulum of height h.



$$h = \frac{R}{\tan 30°}$$

$$\text{Then } T = 2\pi\sqrt{\frac{15\sqrt{3}}{32}}$$

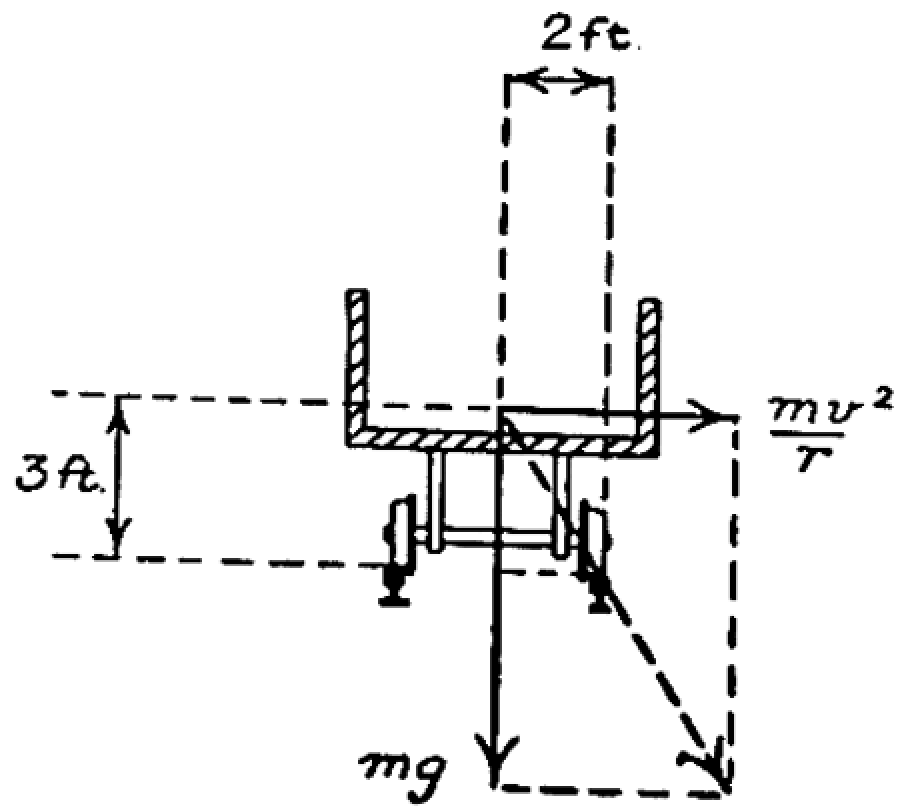
$$= \underline{5{\cdot}66 \text{ sec.}}$$

21.

A railway truck has its centre of gravity at a height of 3 ft. above the rails, which are 4 ft. apart. What would be the maximum safe speed at which it could travel round an unbanked curve of radius 120 ft.?

The forces acting on the truck in a plane at right angles to the rails are its weight mg, the centrifugal force $\frac{mv^2}{r}$ acting horizontally due to its motion round the curve of radius r, and the reactions of the rails.

If the resultant of the first two forces passes above the outer rail, the truck will topple over. The limiting safe case is shown in the diagram, where the resultant passes through the rail.



From similar triangles:

$$\frac{\frac{mv^2}{r}}{mg}=\frac{2}{3}$$

$$\frac{v^2}{rg}=\frac{2}{3}$$

$$\frac{v^2}{120\times 32}=\frac{2}{3}$$

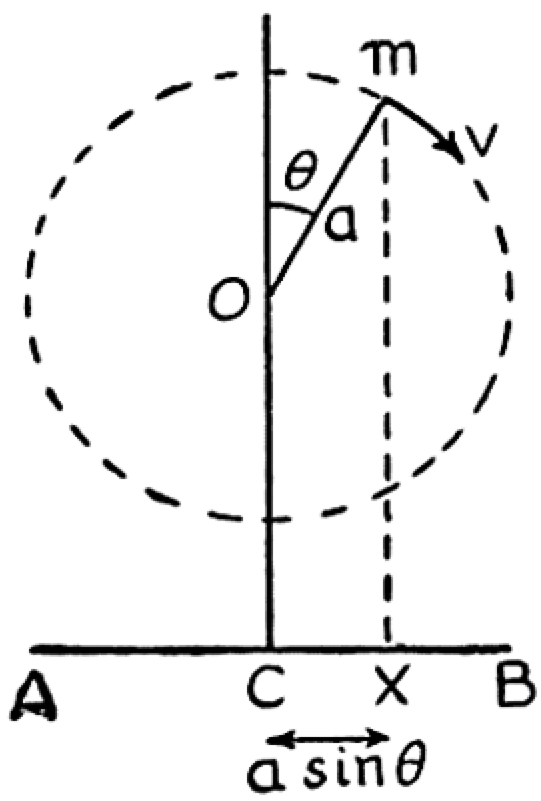
$$\therefore \qquad v=\sqrt{64\times 40}$$
$$=8\times 6{\cdot}32$$
$$=50{\cdot}6 \text{ ft./sec.}$$

To avoid toppling over, the speed of the truck must not exceed 50·6 ft. per sec.

22.

A particle of mass 2 gm. executes simple harmonic vibrations of frequency 10 per second and amplitude 0·5 cm. What is the maximum value of the restoring force?

An important property of simple harmonic motion is that the acceleration is proportional to the displacement from the position of rest.



If the particle X is vibrating between A and B about the mean position C, the amplitude is CB. Call this a. Above AB is drawn a circle of radius a round which the imaginary generating particle may be thought to revolve with uniform speed v. Its radial acceleration, directed towards O is $\frac{v^2}{a}$ and the component of this parallel to AB is $\frac{v^2}{a}\sin\theta$. This is therefore the acceleration of X, directed towards C when its displacement is a sin θ.

So the ratio $\frac{\text{acceleration}}{\text{displacement}}=\frac{v^2}{a^2}$

(or ω^2 where ω is the angular velocity).

$$\text{The frequency of vibration}=\frac{v}{2\pi a}=\frac{1}{2\pi}\sqrt{\frac{\text{acceleration}}{\text{displacement}}}$$
$$=10$$

so
$$\frac{\text{accel.}}{\text{displ.}}=400\pi^2$$

The maximum acceleration occurs when the displacement is $a=0{\cdot}5$ cm. and is therefore equal to $400\pi^2\times 0{\cdot}5=1973{\cdot}9$ cm./sec.^{2} Since force=mass × acceleration, the maximum restoring force is

$$2\times 1973{\cdot}9=\underline{3948 \text{ dynes}}$$

23.

A pendulum consists of a long thread carrying a small lead sphere at its lower end. When set swinging with small amplitude, 20 vibrations occur in 3 minutes. When the thread is shortened by 97 cm., 20 vibrations occupy 2 min. 55·6 sec. Find the original length of the thread and the value of g.

Let the original length be L cm. Then the periodic time in the first case is

$$\frac{3\times 60}{20}=9{\cdot}0 \text{ sec.}=2\pi\sqrt{\frac{L}{g}} \quad \ldots\ldots\ldots\ldots \quad (1)$$

and in the second case

$$\frac{(2\times 60)+55{\cdot}6}{20}=8{\cdot}78 \text{ sec.}=2\pi\sqrt{\frac{L-97}{g}} \quad \ldots\ldots \quad (2)$$

Squaring both sides of both equations and subtracting (2) from (1) gives

$$9^2-8{\cdot}78^2=\frac{4\pi^2L}{g}-\frac{4\pi^2(L-97)}{g}$$

$$\therefore \quad (9+8{\cdot}78)(9-8{\cdot}78)=\frac{4\pi^2\times 97}{g}$$

$$17{\cdot}78\times 0{\cdot}22=\frac{388\pi^2}{g}$$

$$g=\frac{388\times 3{\cdot}142^2}{3{\cdot}912}$$

$$=\underline{979 \text{ cm. per sec.}^2}$$

By substituting for g in equation (1) above we obtain

$$9{\cdot}0=2\pi\sqrt{\frac{L}{979}}$$

whence

$$L=\frac{81\times979}{4\pi^2}$$
$$=\underline{2008\text{ cm.}}$$

24.

A 5 gm. weight is suspended from the end of a spiral spring of negligible mass, and it is found that the addition of further weights, 0·5 gm. at a time, produces uniform extensions of the spring at the rate of 1 cm. for each 0·5 gm. If the additional weights are now removed, what will be the frequency of vertical vibrations of the 5 gm. weight?

The figures given for the static extension of the spring show that the restoring force due to its elasticity is proportional to the extension. The vibrations are therefore simple harmonic and we may apply the standard expression for their frequency

$$n=\frac{1}{2\pi}\sqrt{\frac{\text{restoring force per unit displacement}}{\text{mass}}}$$

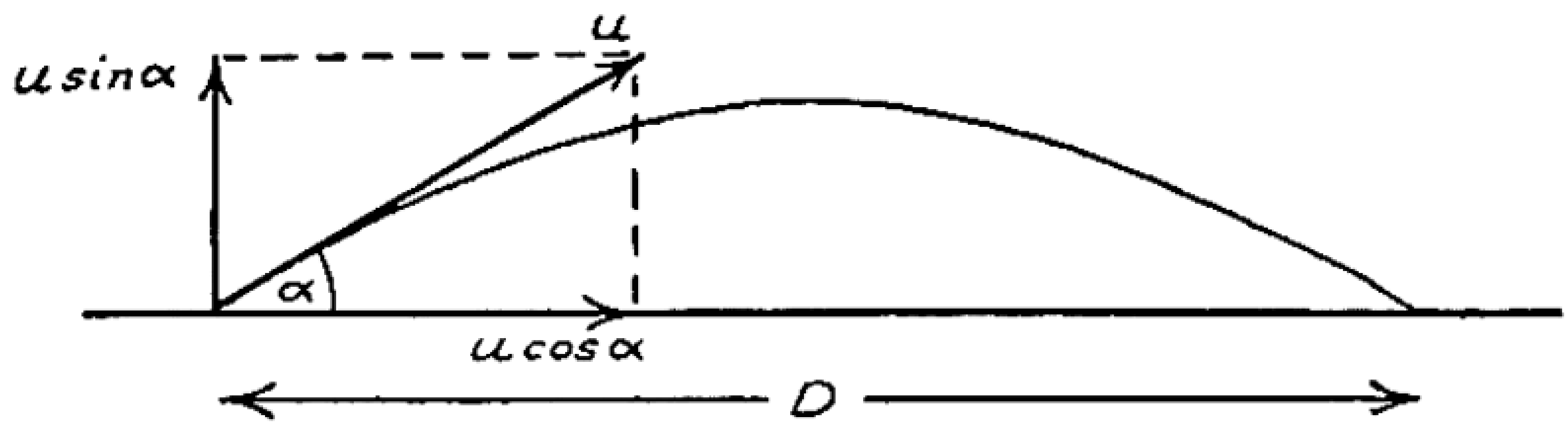
The numerator is the force to extend the spring 1 cm., namely 0·5 gm. wt. or $0{\cdot}5\times981$ dynes.

so

$$n=\frac{1}{2\pi}\sqrt{\frac{0{\cdot}5\times981}{5}}$$
$$=\frac{\sqrt{98{\cdot}1}}{2\times3{\cdot}142}$$
$$=\frac{9{\cdot}904}{6{\cdot}284}$$
$$=\underline{1{\cdot}58\text{ per sec.}}$$

25.

A shell leaves a gun with a velocity of 1500 ft. per sec. At what range will it strike the ground if it is fired at an angle of 30° to the horizontal? (Neglect air resistance.)



The velocity of the shell at any point in its trajectory may be resolved into two components, one vertical, the other horizontal. Of these, the latter remains constant, since there are no horizontal forces acting on the shell. The former is affected by gravity, just as if the shell had been fired vertically with a velocity equal to the vertical component of its muzzle velocity.

Referring to the diagram, if t is the time for which the shell is in the air,

$$D=ut\cos a$$

t may be evaluated by considering the vertical component of u.

From $\quad v=u+at$

$0=u\sin a-g\frac{t}{2}$ for the first half of the trajectory,

which gives $\quad t=\dfrac{2u\sin a}{g}$

$$\therefore\quad D=u\cos a\times\frac{2u\sin a}{g}$$

$$=\frac{2u^2\cos a\sin a}{g}=\frac{u^2}{g}\sin 2a$$

$$=\frac{(1500)^2}{32}\sin 60°$$

$$=\frac{2{\cdot}25\times10^6}{32}\times\frac{\sqrt{3}}{2}\text{ ft.}$$

$$=\frac{2{\cdot}25\times10^6\times\sqrt{3}}{5280\times64}\text{ miles}$$

$$=\underline{11{\cdot}53\text{ miles}}$$

26.

What must be the minimum muzzle velocity for a shell to have a range of 10 miles, neglecting air resistance?

Referring to the previous example, the range is equal to

$$\frac{u^2}{g} \sin 2a$$

where u is the muzzle velocity and a is the angle of projection.

For a given value of u the maximum range will be given when $\sin 2a$ is a maximum, i.e. when $2a=90°$ and $a=45°$.

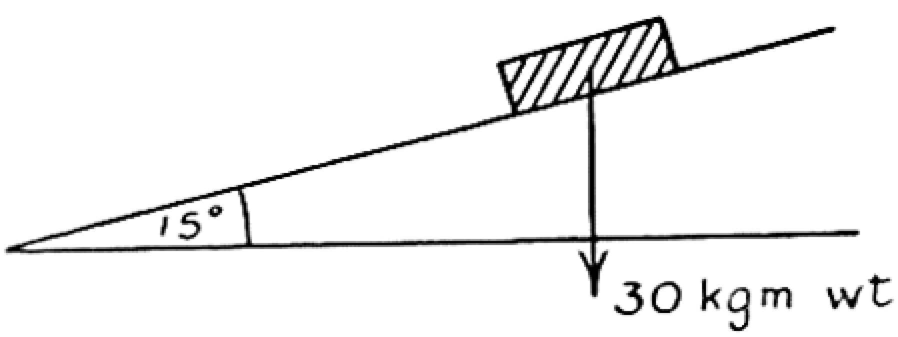
Then $$5280 \times 10 = \frac{u^2}{32}$$

$\therefore$ $$u = \sqrt{32 \times 5280 \times 10} \text{ ft./sec.}$$
$$= 1300 \text{ ft./sec.}$$

The minimum muzzle velocity for a range of 10 miles is 1300 ft./sec., the shell being projected at an angle of 45° with the horizontal.

27.

How much work is done in hauling a load of 30 kgms. a distance of 10 metres up a slope inclined at 15° to the horizontal?



For a distance of 10 metres travelled by the load along the slope, the vertical distance through which it has risen is

$(10 \times \sin 15°)$ metres

and the work done against gravity is

$$(30 \times 10 \times \sin 15°) \text{ kgm. metres}$$
$$= (300 \sin 15° \times 1000 \times 100) \text{ gm. cm.}$$
$$= (300 \sin 15° \times 10^5 \times 981) \text{ dyne cm. or ergs}$$
$$= (300 \times 0{\cdot}2588 \times 10^5 \times 981) \text{ ergs}$$
$$= 7{\cdot}62 \times 10^9 \text{ ergs}$$

From a slightly different point of view:

The component of the weight of the load parallel to the plane

$$= (30 \sin 15°) \text{ kgm. wt.}$$
$$= (30 \times 1000 \times \sin 15°) \text{ gm. wt.}$$
$$= (30 \times 1000 \times 981 \times \sin 15°) \text{ dynes}$$

The hauling force must be equal and opposite to this, and the work done when such a force has its point of application moved through 10 metres

$$=(30\times1000\times981\times\sin 15^\circ\times1000) \text{ ergs}$$
$$=\underline{7{\cdot}62\times10^9 \text{ ergs}}$$

28.

Calculate the horse-power developed by a locomotive in hauling a train weighing 500 tons up an incline of 1 in 100 at a speed of 45 miles per hour.

$$45 \text{ m.p.h.}=\frac{45\times5280}{60\times60} \text{ ft. per sec.}$$
$$=66 \text{ ft. per sec.}$$

For every 100 ft. along the track the train is raised a vertical distance of 1 ft. Therefore the vertical component of the speed of the train

$$=\frac{66}{100} \text{ ft. per sec.}$$

The amount of work done in raising 500 tons 0·66 ft. against gravity

$$=(500\times2240\times0{\cdot}66) \text{ ft. lb.}$$

Rate of working $=(500\times2240\times0{\cdot}66)$ ft. lb. per sec.

$$=\frac{500\times2240\times0{\cdot}66}{550} \text{ horse-power}$$
$$=\underline{1344 \text{ horse-power}}$$

29.

A car weighing half a ton accelerates uniformly from rest, taking 30 sec. to attain a speed of 40 miles per hour. Neglecting frictional losses, calculate the horse-power of the engine when the speed is (*a*) 20 m.p.h. and (*b*) 30 m.p.h.

When converting speeds in m.p.h. to ft. per sec., it is convenient to remember that 60 m.p.h.=88 ft. per sec.

So $\quad 40 \text{ m.p.h.}=\frac{2}{3}\times88 \text{ ft. per sec.}$

$$20 \text{ m.p.h.} = \frac{1}{3} \times 88 \text{ ft. per sec.}$$

$$30 \text{ m.p.h.} = \frac{1}{2} \times 88 \text{ ft. per sec.}$$

Since a speed of $\frac{2}{3} \times 88$ ft. per sec. is attained in 30 sec. from rest, the acceleration a is equal to

$$\frac{2 \times 88}{3 \times 30} \text{ ft. per sec. per sec.}$$

$$= \frac{88}{45} \text{ ft./sec.}^2$$

The force F which produces an acceleration a in a mass m is given by:

$$F = ma$$

$$\therefore \quad F = 1120 \times \frac{88}{45} \text{ poundals}$$

$$= \frac{1120 \times 88}{32 \times 45} \text{ lb. wt.}$$

This force will be constant so long as the acceleration is constant.

Since Power = rate of doing work
= work done per sec.
= force × velocity

Therefore (*a*), the power developed at 20 m.p.h.

$$= \frac{1120 \times 88}{32 \times 45} \cdot \frac{88}{3} \text{ ft. lb. per sec.}$$

$$= \frac{1120 \times 88 \times 88}{32 \times 45 \times 3 \times 550} \text{ horse-power}$$

$$= \underline{3{\cdot}65 \text{ horse-power}}$$

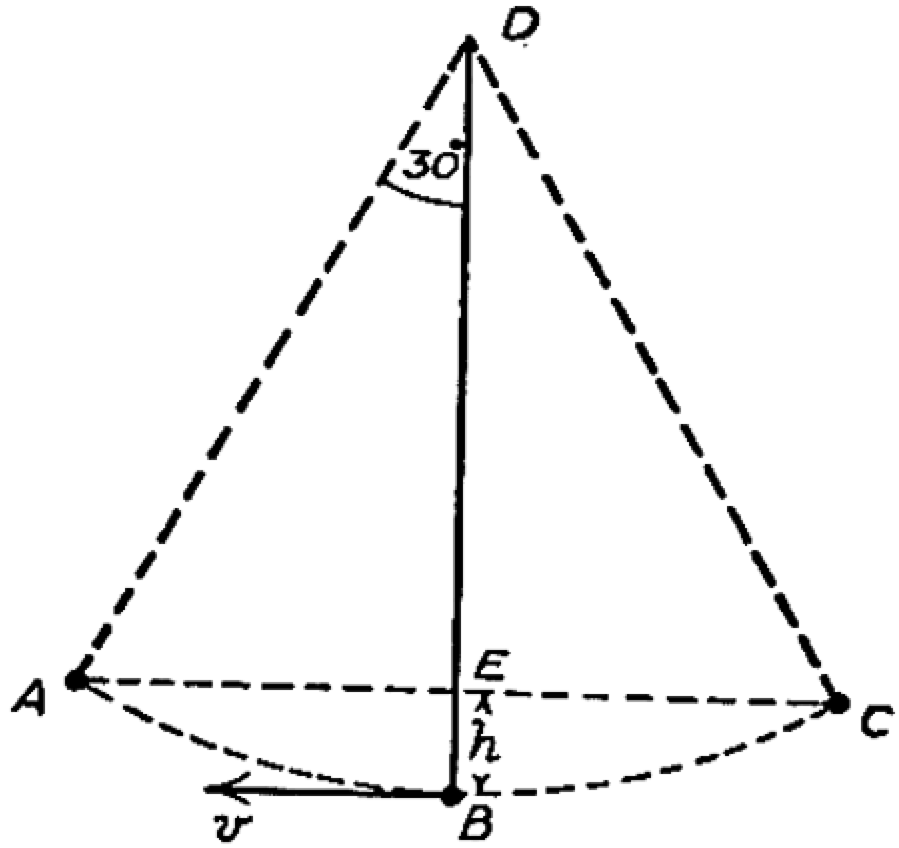
and (*b*), the power developed at 30 m.p.h.

$$= \frac{1120 \times 88 \times 88}{32 \times 45 \times 2 \times 550} \text{ horse-power}$$

$$= \underline{5{\cdot}48 \text{ horse-power}}$$

30.

A simple pendulum of length 1 metre is vibrating with an amplitude of 30°. What is the velocity of the pendulum bob at its lowest point?



By the principle of the conservation of energy, the kinetic energy possessed by the bob at its lowest point B must equal its potential energy at A or C with respect to B.

$$\therefore \quad \frac{1}{2}mv^2 = mgh$$

or $$v = \sqrt{2gh}$$

$$h = BD - ED$$

$$= 100 - 100 \cos 30°$$

since ADC is an equilateral triangle.

$$= 100\left(1 - \frac{\sqrt{3}}{2}\right)$$

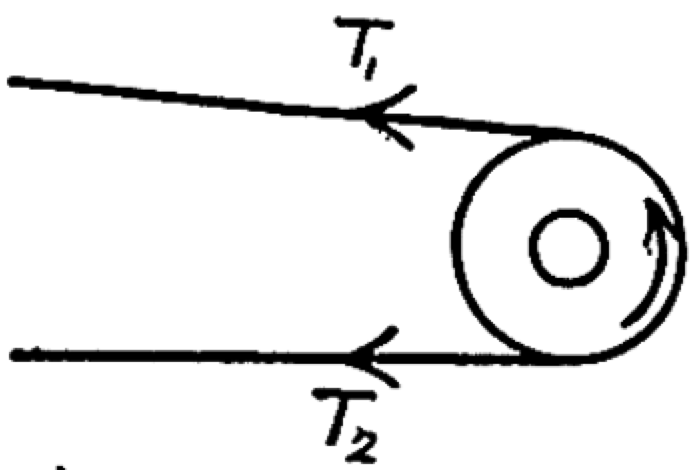
$$= 13{\cdot}4 \text{ cm.}$$

$$\therefore \quad v = \sqrt{2 \times 981 \times 13{\cdot}4} \text{ cm. per sec.}$$

$$= \underline{162{\cdot}1 \text{ cm. per sec.}}$$

31.

A belt-drive transmits 10 horse-power to a pulley 18 in. in diameter. What is the difference between the tensions in the two straight portions of the belt if the pulley rotates twice per second?



Let the tensions be T_1 and T_2 lb. wt.

The difference between them, $(T_1 - T_2)$, is the frictional force acting tangentially on the rim of the pulley.

The work done by this force in turning the pulley through 1 revolution is

$$1{\cdot}5\pi(T_1 - T_2) \text{ ft. lb.}$$

and the work done per second is therefore

$$2 \times 1{\cdot}5\pi(T_1 - T_2) \text{ ft. lb.}$$

Since $\quad 1 \text{ horse-power} = 550 \text{ ft. lb. per sec.}$

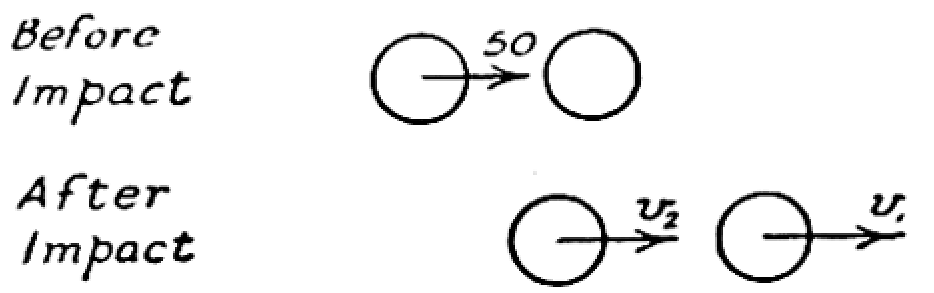
$$\therefore \qquad 10 = \frac{3\pi(T_1 - T_2)}{550}$$

$$T_1 - T_2 = \frac{5500}{3\pi} \text{ lb. wt.}$$

$$= \underline{584 \text{ lb. wt.}}$$

32.

Two similar balls, each of mass 20 gm., lie some distance apart in a smooth horizontal groove. One is projected towards the other with a speed of 50 cm. per sec. If the coefficient of restitution between the balls is 0·75, calculate the velocities of the balls after impact, and the loss of energy in the system during impact.



Let the velocities after impact be v_1 for the ball initially at rest and v_2 for the other.

The coefficient of restitution is the ratio of the relative velocity after impact to that before impact.

$$\therefore \qquad 0{\cdot}75 = \frac{v_1 - v_2}{50}$$

The total momentum before impact must be the same as after.

$$\therefore \qquad 50m = mv_1 + mv_2 \text{ or } 50 = v_1 + v_2$$

all the signs being positive because all the velocities are in the same direction.

Substituting for v_1 in the first equation:

$$0{\cdot}75 = \frac{(50 - v_2) - v_2}{50}$$

$$37{\cdot}5 = 50 - 2v_2$$

$$v_2 = \underline{6{\cdot}25 \text{ cm./sec.}}$$

$$v_1 = \underline{43{\cdot}75 \text{ cm./sec.}}$$

Initial kinetic energy of the system $= \frac{1}{2}mv^2$

$$= \frac{1}{2} \times 20 \times 50^2$$

$$= 25{,}000 \text{ ergs}$$

$$\text{Final kinetic energy} = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$$
$$= \frac{1}{2} \times 20(6{\cdot}25^2 + 43{\cdot}75^2)$$
$$= 10(39{\cdot}1 + 1914)$$
$$= 19{,}531 \text{ ergs}$$

$$\text{Loss of energy during impact} = 25{,}000 - 19{,}531 = \underline{5469 \text{ ergs}}$$

33.

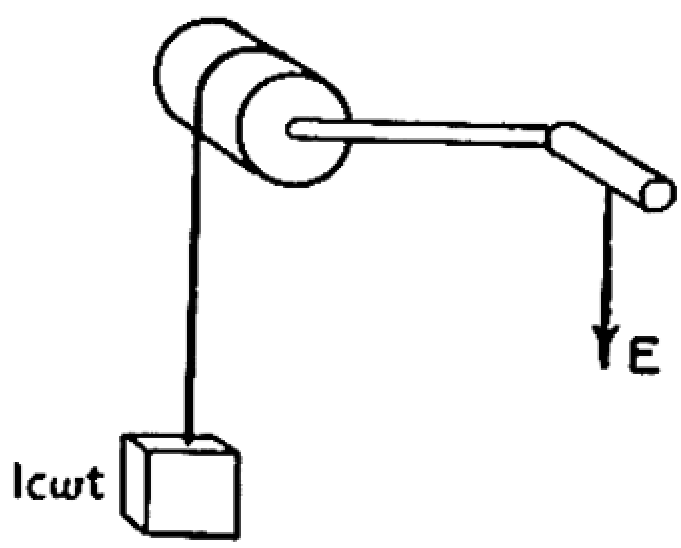
A windlass with an efficiency of 85 per cent. is used for raising a load of 1 cwt. If the diameter of the drum on which the rope is wound is 6 in. and the length of the crank arm is 18 in., what force must be applied to the crank arm to raise the load?

The efficiency of a machine is equal to

$$\left(\frac{\text{Mechanical Advantage}}{\text{Velocity Ratio}} \times 100\right) \text{ per cent.}$$

In a windlass,

$$\text{Velocity Ratio} = \frac{\text{length of crank arm}}{\text{radius of drum}}$$
$$= \frac{18}{3} = 6$$



$$\text{Mechanical Advantage} = \frac{\text{load}}{\text{effort}}$$
$$= \frac{112}{E}$$

where E is the effort in pounds weight.

$$\therefore \qquad 85 = \frac{112}{6E} \times 100$$
$$E = \frac{11200}{6 \times 85}$$
$$= \underline{22{\cdot}0 \text{ lb. wt.}}$$

34.

A screw-jack has a screw of pitch $\frac{1}{2}$ in. turned by a handle having an arm 2 ft. long. If the efficiency of the screw-jack is 40 per cent., what is the greatest load which can be raised on it by a man capable of exerting an effort of 50 lb. wt. on the end of the handle?

$$\text{Efficiency of a machine} = \left(\frac{\text{Mechanical Advantage}}{\text{Velocity Ratio}} \times 100\right) \text{ per cent.}$$

If L stands for the maximum load in pounds,

$$\text{Mechanical Advantage} = \frac{\text{load}}{\text{effort}} = \frac{L}{50}$$

$$\text{Velocity Ratio} = \frac{\text{Distance moved through by effort in 1 revn.}}{\text{Pitch of screw}}$$

$$= \frac{2\pi \times 2 \times 12}{0{\cdot}5}$$

$$= 96\pi$$

$$\therefore \qquad 40 = \frac{L \times 100}{50 \times 96\pi}$$

$$L = \frac{40 \times 50 \times 96\pi}{100}$$

$$= 6033 \text{ lb.}$$

$$= \underline{2 \text{ tons } 13 \text{ cwt. } 97 \text{ lb.}}$$

35.

The mass of 1 cu. ft. of water is 62·3 lb. The specific gravities of paraffin oil and lead are 0·80 and 11·4 respectively. Give the densities of these two substances in both British and metric units.

Recalling the definitions of specific gravity and density, we see that if ρ_w is the density of water and ρ_s the density of a substance of specific gravity s, then

$$\rho_s = s\rho_w$$

Since in the *metric system* the density of water is 1 gm./c.c., therefore

$$\rho_s = s \text{ gm./c.c.}$$

that is, density and specific gravity are numerically equal.

Therefore the densities of lead and paraffin oil are <u>11·4 gm./c.c. and 0·80 gm./c.c. respectively.</u>

But in the *British system*, the density of water being 62·3 lb./cu. ft., we have

$$\rho_s = (62{\cdot}3 \times s) \text{ lb./cu. ft.}$$

Whence the required densities are:

Paraffin oil $\rho = (62{\cdot}3 \times 0{\cdot}8)$ lb./cu. ft.

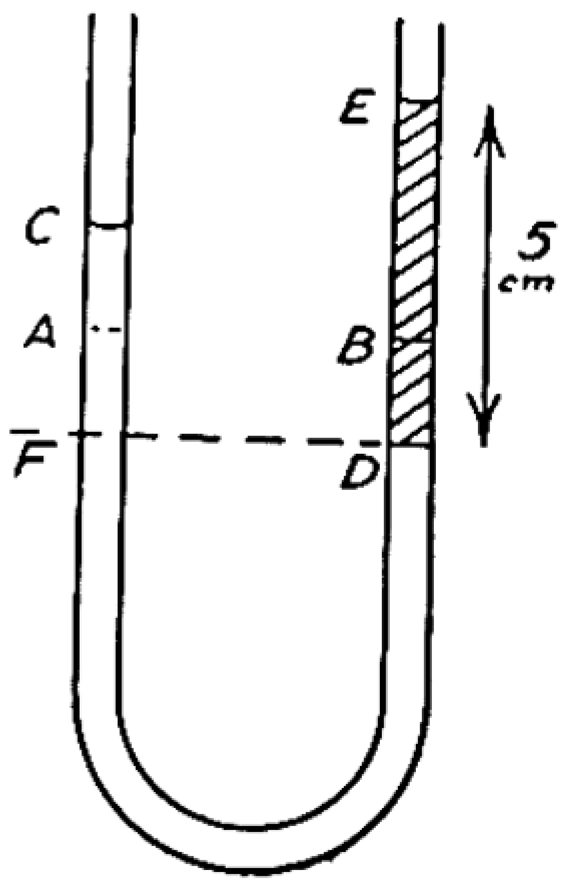
$= \underline{49{\cdot}8 \text{ lb./cu. ft.}}$

Lead $\rho = (62{\cdot}3 \times 11{\cdot}4)$ lb./cu. ft.

$= \underline{710{\cdot}2 \text{ lb./cu. ft.}}$

36.

A U-tube with its limbs vertical is about half filled with water. Paraffin oil of specific gravity 0·8 is then poured into one of the limbs to a depth of 5 cm. How far will the water rise in the other limb?



In the diagram AB represents the original water level, ED is the column of oil 5 cm. long, and C is the final water level in the other limb.

Since the tube may be assumed of uniform bore, the length of water column must be constant, and therefore AC = BD.

By the laws of hydrostatics, the pressures at D and F are equal, so that the column of water CF exerts the same pressure as the column of oil ED.

$$\therefore \quad CF \times 1 = 5 \times 0{\cdot}8$$
$$CF = 4 \text{ cm.}$$

and, since $CA = \frac{1}{2}CF$, $CA = 2$ cm.

<u>The water will rise 2 cm. in the limb of the U-tube.</u>

37.

A piece of metal weighs 178·5 gm. in air and 157·5 gm. in water. What is its density?

The solution of this problem involves the use of Archimedes' Principle, whereby we may equate the loss in weight of the metal to the weight of water it displaces.

Let the volume of the metal be V c.c.
The loss in weight $=(178{\cdot}5-157{\cdot}5)=21{\cdot}0$ gm.
The volume of water displaced $=V$ c.c.
The weight of water displaced $=V$ gm.

$\therefore \quad 21{\cdot}0=V$

The volume of the metal is 21·0 c.c. and its density

$$=\frac{178{\cdot}5}{21} \text{ gm./c.c.}$$
$$=\underline{8{\cdot}5 \text{ gm./c.c.}}$$

38.

A cubical block of wood whose edges are 1 ft. long floats (*a*) in water and (*b*) in paraffin oil. What depth of wood is below the surface of the liquid in each case? The densities of wood and paraffin oil are 0·6 and 0·8 gm. per c.c. respectively.

Considering the general case—if ρ_s and ρ_L are the densities of a floating body and of the liquid in which it is floating, and if V is the volume of the body and v the volume immersed,
the loss in weight of the body $=\rho_s V-0$
since its weight when floating is zero.

The weight of liquid displaced $=\rho_L v$
and by Archimedes' Principle $\rho_s V=\rho_L v$

or
$$\frac{v}{V}=\frac{\rho_s}{\rho_L}$$

a result which is easy to remember: The ratio of the volume immersed to the total volume equals the ratio of the density of the solid to that of the liquid.

Assuming that the cube floats with one pair of faces parallel to the liquid surface, and letting d stand for the depth immersed:

$$(a) \quad \frac{d}{12}=\frac{0{\cdot}6}{1} \qquad \therefore \quad \underline{d=7{\cdot}2 \text{ in.}}$$

$$(b) \quad \frac{d}{12}=\frac{0{\cdot}6}{0{\cdot}8} \qquad \therefore \quad \underline{d=9 \text{ in.}}$$

39.

A piece of glass tubing 30 cm. long, sealed at both ends, weighs 15 gm. in air and just floats, completely submerged, in water. If the density of glass is 2·5 gm./c.c., what are the internal and external radii of the tube?

Let the internal and external radii be r and R cm. respectively.
By Archimedes' Principle:

$$\text{Loss in weight} = \text{Weight of water displaced}$$
$$15-0=30\times \pi R^2$$

(When floating, its weight is zero.)

$$\therefore \quad R^2=\frac{1}{2\pi}$$
$$\underline{R=0{\cdot}399 \text{ cm.}}$$

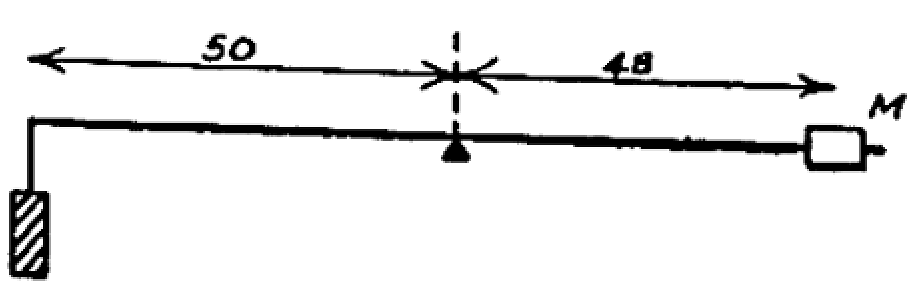
$$\text{Mass of glass}=2{\cdot}5\times 30\times \pi(R^2-r^2)$$
$$=15 \text{ gm.}$$

$$\therefore \quad 75\times\left(\frac{1}{2\pi}-r^2\right)=15$$

$$r^2=\frac{1}{2\pi}-\frac{1}{5\pi}=\frac{5-2}{10\pi}$$
$$=\frac{3}{31{\cdot}42}$$
$$\underline{r=0{\cdot}309 \text{ cm.}}$$

40.

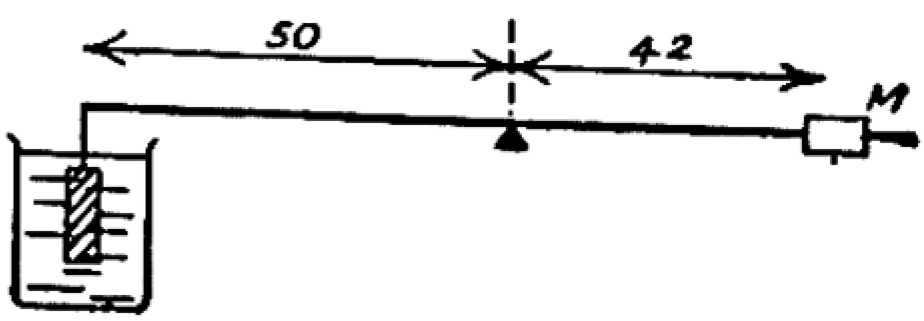
A metre rule supported by a knife-edge at its mid-point has a piece of metal hung at one end and a weight capable of sliding along the rule. It is found that the rule balances horizontally when the weight is 48 cm. from the knife-edge. When the piece of metal is totally immersed in water, the weight has to be moved 6 cm. nearer to the knife-edge to restore balance. Calculate the specific gravity of the metal.



Let the volume of the metal be V c.c. and its specific gravity s. Let the mass of the weight be M gm.

Then, taking moments about the point of balance:

$$50Vs=48M$$



By Archimedes' Principle, when the metal is immersed in water, its loss in weight is V gm. and its weight in water $(Vs-V)$ gm.

Taking moments again about the knife-edge:

$$50(Vs-V)=(48-6)M$$

Dividing this equation by the first:

$$\frac{Vs-V}{Vs}=\frac{42}{48}$$

$$\therefore \quad \frac{s-1}{s}=\frac{7}{8}$$

$$s=8$$

The specific gravity of the metal is equal to 8.

41.

A Nicholson's hydrometer of mass 150 gm. needs an addition of 12·5 gm. to sink it in water to the standard mark on the stem. In brine, a further 13·0 gm. are needed to sink it to the mark. What is the specific gravity of the brine?

Suppose the hydrometer displaces V c.c. of liquid when immersed to the mark.

Then the weight of water displaced $=V$ gm.

The loss in weight in water $=(150+12\cdot5)$ gm.
since the weight of a floating body is zero.

$\therefore$ By Archimedes' Principle $\quad V=162\cdot5$

Let the specific gravity of the brine be S.

Then the weight of brine displaced $=(162\cdot5\times S)$ gm.

The loss in weight in brine $=(150+12\cdot5+13)$ gm.

$\therefore$ By Archimedes' Principle

$$162\cdot5\times S=175\cdot5$$

$$S=\frac{175\cdot5}{162\cdot5}$$

$$=1\cdot08$$

The specific gravity of the brine is 1·08.

42.

A Nicholson's hydrometer sinks in water to the standard mark when 11·20 gm. are placed in the upper pan. When a piece of wood is placed in the upper pan, the weight required to sink the hydrometer to the mark is 8·93 gm. When the wood is tied to the lower pan with fine thread 12·43 gm. are required to sink to the mark. What is the density of the wood?

The Nicholson's hydrometer is here being used as a balance to weigh the wood in air, in the upper pan, and in water, in the lower pan.

Weight of wood in air $=11{\cdot}20-8{\cdot}93=2{\cdot}27$ gm.

Weight of wood in water $=11{\cdot}20-12{\cdot}43=-1{\cdot}23$ gm.

Loss in weight $=2{\cdot}27-(-1{\cdot}23)=3{\cdot}50$ gm.

By Archimedes' Principle:

Weight of water displaced $=3{\cdot}50$ gm.

Volume of water displaced $=3{\cdot}50$ c.c.

Volume of wood $=3{\cdot}50$ c.c.

Density of wood $=\dfrac{\text{mass}}{\text{volume}}=\dfrac{\text{weight in air}}{\text{volume}}=\dfrac{2{\cdot}27}{3{\cdot}5}\ \dfrac{\text{gm. per}}{\text{c.c.}}$

$=\underline{0{\cdot}65 \text{ gm. per c.c.}}$

43.

A hydrometer consisting of a bulb of volume 5 c.c. surmounted by a cylindrical stem of diameter 5 mm. sinks in water to a depth of 2 cm. above the bulb. To what depth will it sink in a liquid of specific gravity 0·95?

As with all floating bodies, the weight of liquid displaced by the hydrometer equals the weight of the hydrometer in air. So the product of the volume immersed and the density of the liquid is constant for a given hydrometer.

In water, the volume immersed is the volume of the bulb plus the volume of 2 cm. of the stem

$$=5+(2\pi\times0{\cdot}25^2) \text{ c.c.}$$

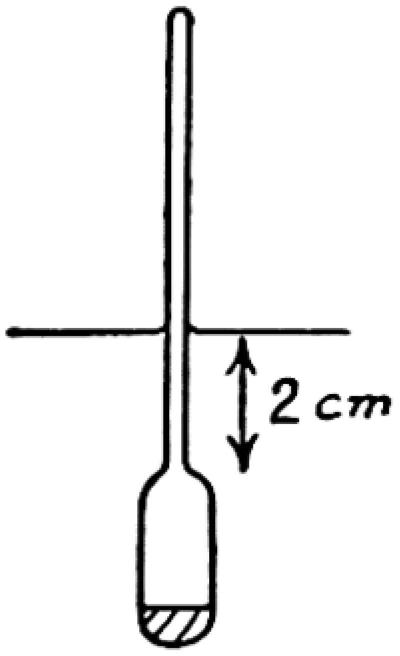
In the liquid of specific gravity 0·95 the volume immersed

$$=5+(l\pi\times0{\cdot}25^2)\text{ c.c.}$$

where l is the length of stem immersed.

$$\therefore\quad 5+(2\pi\times0{\cdot}25^2)=0{\cdot}95[5+(l\pi\times0{\cdot}25^2)]$$

$$5(1-0{\cdot}95)=0{\cdot}0625\pi(0{\cdot}95l-2)$$



$$\frac{0{\cdot}25}{0{\cdot}0625\pi}=0{\cdot}95l-2$$

$$\therefore\quad l=\frac{1}{0{\cdot}95}\left(\frac{1}{0{\cdot}25\pi}+2\right)$$

$$=\frac{0{\cdot}5\pi+1}{0{\cdot}95\times0{\cdot}25\pi}$$

$$=\frac{2{\cdot}571}{0{\cdot}95\times0{\cdot}785}$$

$$=\underline{3{\cdot}45\text{ cm.}}$$

The hydrometer will sink to a depth of <u>3·45 cm. above the bulb</u> in the liquid of specific gravity 0·95.

44.

What load could just be lifted off the ground by a spherical balloon 2 metres in diameter containing hydrogen at atmospheric pressure if the mass of the envelope and fittings is 100 gm.?

Density of hydrogen=0·090 gm. per litre; density of air=1·29 gm. per litre.

Suppose that a load of m gm. can just be lifted off the ground.

In applying Archimides' Principle to a balloon problem, it must be borne in mind that when the balloon is floating in equilibrium it weighs nothing, and its loss in weight is therefore equal to the total weight of balloon, load and contained gas.

So by Archimedes' Principle:

(Weight of balloon and load)+(Weight of gas)

=(Weight of air displaced)

If the volume of the balloon be V and the densities of hydrogen and air ρ_h and ρ_a, then

(Weight of balloon and load)+ $V\rho_h=V\rho_a$

$\therefore$ (Weight of balloon and load) $=V(\rho_a-\rho_h)$

Thus the total lifting capacity of the balloon is equal to the product of its volume and the difference between the densities of the gases inside and outside.

In this problem then:

$$100+m=\left(\frac{4}{3}\times\frac{22}{7}\times 100^3\right)(0{\cdot}00129-0{\cdot}00009)$$

$$=\frac{88}{21}\times 10^6\times 0{\cdot}00120$$

$$=5029$$

$$\therefore \qquad \underline{m=4929 \text{ gm.}}$$

45.

A captive balloon contains 400,000 litres of hydrogen. If the mass of the envelope, rigging, etc., is 100 kgm., how high would the balloon ascend if the cable, of mass 50 gm. per metre, by which it is secured, remains vertical and the pressure inside the balloon, by the operation of a valve, is always equal to that of the surrounding atmosphere? Assume that the temperature is 0° C. at all altitudes, and that atmospheric pressure is equal to 76 cm. of mercury at ground level, decreasing by 7·6 cm. for each kilometre increase in altitude.

The densities of air and hydrogen at N.T.P. are 1·29 and 0·090 gm. per litre respectively.

From Ex. 40, we may write:

Total lift of balloon $=V(\rho_a-\rho_h)$, where V is the volume of the balloon and ρ_h and ρ_a are the densities of the hydrogen within the balloon and the air surrounding it.

If h is the height in kilometres reached by the balloon, the pressure at this height is $(76-7{\cdot}6h)$ cm., and the densities are therefore:

$$\rho_a=1{\cdot}29\times\frac{(76-7{\cdot}6h)}{76}=1{\cdot}29\left(1-\frac{h}{10}\right) \text{ gm. per litre.}$$

$$\rho_h=0{\cdot}09\times\frac{(76-7{\cdot}6h)}{76}=0{\cdot}09\left(1-\frac{h}{10}\right) \text{ gm. per litre.}$$

Therefore

$$\text{the total lift}=400{,}000\left[1{\cdot}29\left(1-\frac{h}{10}\right)-0{\cdot}09\ \ 1-\frac{h}{10}\right] \text{ gm.}$$

$$=400{,}000\times 1{\cdot}20\times\left(1-\frac{h}{10}\right) \text{ gm.}$$

$$=480\times\left(1-\frac{h}{10}\right) \text{ kgm.}$$

As the balloon is in equilibrium, this must equal the weight of the envelope, etc., plus the weight of $1000h$ metres of cable.

$$\therefore \qquad 100+\left(\frac{50\times 1000h}{1000}\right)=480\times\left(1-\frac{h}{10}\right)$$

$$100+50h=480-48h$$

$$h=\frac{380}{98}$$

$$=\underline{3{\cdot}88 \text{ km.}}$$

46.

The weight in air of a piece of cork obtained with a reliable balance using brass weights is 45·75 gm. What is the true mass of the cork?

Density of brass $=8{\cdot}5$ gm. per c.c.
Density of cork $=0{\cdot}25$ gm. per c.c.
Density of air $=0{\cdot}00129$ gm. per c.c.

Owing to the buoyancy of the air, the weights in air of the cork and the brass are less than they would be in a vacuum.

By Archimedes' Principle, the loss in weight is equal to the weight of air displaced.

Volume of brass $=\dfrac{45{\cdot}75}{8{\cdot}5}$ c.c.

$=$volume of air displaced by brass

$\therefore$ Weight of air displaced by brass

$$=\left(\frac{45{\cdot}75}{8{\cdot}5}\times 0.00129\right) \text{ gm.}$$

$=$loss in weight of brass

$\therefore$ Weight of brass in air

$$=45{\cdot}75-\left(\frac{45{\cdot}75}{8{\cdot}5}\times 0{\cdot}00129\right) \text{ gm.}$$

$$=45{\cdot}75\left(1-\frac{0{\cdot}00129}{8{\cdot}5}\right) \text{ gm.}$$

$=$weight of cork in air

Approx. volume of cork

$$=\frac{45{\cdot}75}{0{\cdot}25}\text{ c.c.}$$

=volume of air displaced by cork

$\therefore$ Weight of air displaced by cork

$$=\left(\frac{45{\cdot}75}{0{\cdot}25}\times 0{\cdot}00129\right)\text{ gm.}$$

=loss in weight of cork

$\therefore$ Weight of cork *in vacuo*

=(weight in air)+(loss in weight)

$$=\left[45{\cdot}7\left(1-\frac{0{\cdot}00129}{8{\cdot}5}\right)\right]+\left(\frac{45{\cdot}75}{0{\cdot}25}\times 0{\cdot}00129\right)$$

$$=45{\cdot}75\left(1-\frac{0{\cdot}00129}{8{\cdot}5}+\frac{0{\cdot}00129}{0{\cdot}25}\right)$$

$$=45{\cdot}75(1+0{\cdot}00511)$$

$$=45{\cdot}75+0{\cdot}234\text{ gm.}$$

$\therefore$ True mass of cork=45·98 gm.

47.

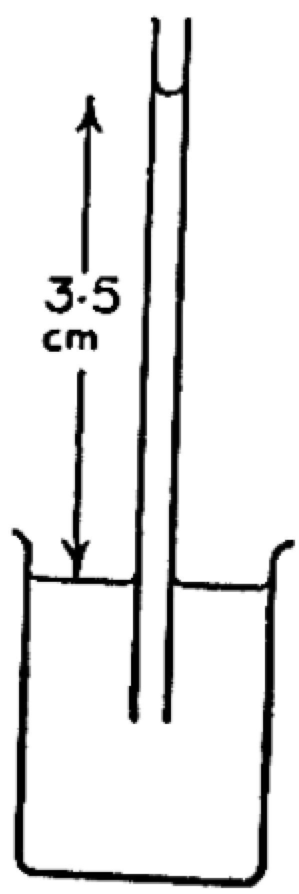
Calculate the radius of a capillary tube if when it is dipped vertically into a beaker of water, the water stands 3·5 cm. higher in the capillary than in the beaker. The surface tension of water is 73 dynes per cm.

Working this from first principles:

The volume of water in the tube above the level of water in the beaker

$$=\pi r^2\times 3{\cdot}5\text{ c.c.}$$

(ignoring a very small correction due to the curved meniscus).



The weight of this column of water

$$=\pi r^2\times 3{\cdot}5\times 981\text{ dynes}$$

This must be equal and opposite to the upward pull due to surface tension around the periphery of the meniscus where it touches the walls of the tube tangentially.

$$\therefore\qquad 2\pi r\times 73=\pi r^2\times 3{\cdot}5\times 981$$

$$2\times 73=r\times 3{\cdot}5\times 981$$

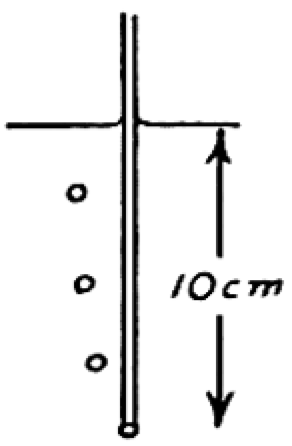
$$r=\underline{0{\cdot}0425\text{ cm.}}$$

48.

Air is bubbled into water through a vertical capillary tube of radius 0·5 mm., the end of which is 10 cm. below the surface. What is the minimum pressure of air required?

Surface tension of water=75 dynes per cm.

(Assume that each bubble breaks away from the end of the tube when its diameter equals that of the tube.)



Pressure in the water 10 cm. below the surface

$$=(10\times 981)\ \text{dynes/cm.}^2$$

Pressure inside bubble of radius r is $\frac{2T}{r}$ in excess of that outside.

$\therefore$ Pressure inside bubble of radius 0·05 cm. at end of tube

$$=\left[\left(\frac{2\times 75)}{0{\cdot}05}+(10\times 981\right)\right]\ \text{dyne/cm.}^2$$

$$=\underline{12{,}810\ \text{dyne/cm.}^2}$$

And this is the minimum pressure required to blow air out of the tube. It may also be expressed as

$$\frac{12{,}810}{981}=\underline{13{\cdot}06\ \text{cm. of water}}$$

or $$\frac{13{\cdot}06}{13{\cdot}6}=\underline{0{\cdot}960\ \text{cm. of mercury}}$$

49.

A capillary tube is attached below the tap of a burette and the times taken for various liquids to run out of the burette from the zero to the 20 c.c. division. The times for water, alcohol and paraffin oil are 12, 15 and 30 seconds. If their respective densities are 1·00, 0·97 and 0·80 gm. per c.c., and the coefficient of viscosity for water at the temperature of the experiment is 0·01 poise, what are the viscosities of the other two liquids?

Poiseuille's equation for the volume of liquid flowing per second through a capillary tube is

$$\frac{V}{t}=\frac{\pi P r^4}{8\eta L}$$

in which V is the volume flowing in t seconds through a tube of radius r and length L under a pressure P, η being the coefficient of viscosity. In this problem r and L are constant and P is proportional to ρ, the density of the liquid. So we may write:

$$\frac{V}{t}=\frac{K\rho}{\eta}$$

where K is a constant

And for equal volumes

$$t=\frac{K'\eta}{\rho}$$

where $$K'=\frac{V}{K}$$

For water

$$12=K'\times\frac{0{\cdot}01}{1};\ K'=\frac{12}{{\cdot}01}=1200$$

And therefore for alcohol

$$15=\frac{1200\eta}{{\cdot}97};\ \underline{\eta=0{\cdot}012 \text{ poise}}$$

and for paraffin

$$30=\frac{1200\eta}{{\cdot}80};\ \underline{\eta=0{\cdot}020 \text{ poise}}$$

50.

Find the maximum velocity reached by a steel ball of radius 1 mm., falling freely in castor oil, by applying Stokes' Law for the retarding force acting on a sphere moving through a viscous fluid: $F=6\pi\eta r V$, where F is the force, V the velocity, r the radius of the sphere, and η the viscosity of the fluid. The density of steel is 7·80 gm. per c.c. and of castor oil 0·97 gm. per c.c. The viscosity of the oil is 15 poise (c.g.s. units).

The resultant vertical force on the sphere is equal to its weight minus the buoyancy of the liquid minus the viscous retarding force. When the velocity has reached its maximum value (called the terminal velocity) this resultant force must be zero.

i.e.

$$W-B-F=O$$

where W is the weight and B the buoyancy.

Now $\quad W=\text{volume}\times\text{density of steel}\times g=\frac{4}{3}\pi r^3\rho_1 g$

$$B=\text{volume}\times\text{density of oil}\times g=\frac{4}{3}\pi r^3\rho_2 g$$

(Archimedes' Principle)

$$F=6\pi\eta r V_t$$

V_t being the terminal velocity.

$$\therefore \quad \frac{4}{3}\pi r^3 g(\rho_1-\rho_2)-6\pi\eta r V_t=0$$

$$\frac{2}{3}r^2 g(\rho_1-\rho_2)=3\eta V_t$$

$$\frac{2}{9}\frac{r^2 g}{\eta}(\rho_1-\rho_2)=V_t$$

$$=\frac{2\times\cdot01\times981\times6\cdot83}{9\times15}$$

$$=\underline{0\cdot993 \text{ cm. per sec.}}$$

51.

A steel wire 3 metres long and 1 mm. in diameter is suspended by one end from a rigid support. By how much will the wire extend if a load of 10 kgm. is suspended from it? How much work has been done in producing this extension? Young's modulus for steel$=2\times10^{12}$ dynes per sq. cm.

Let the extension be e cm.

The stress (force per unit area) due to the addition of 10 kgm.

$$=\frac{10{,}000\times981}{\pi\times0\cdot05^2}\text{ dyne/cm.}^2$$

The strain (extension per unit length)

$$=\frac{e}{300}$$

$$\frac{\text{stress}}{\text{strain}}=2\times10^{12}=\frac{10^4\times981}{\pi\times0{\cdot}0025}\times\frac{300}{e}$$

$$\therefore \qquad e=\frac{3\times981\times10^6}{2\pi\times0{\cdot}0025\times10^{12}}\text{ cm.}$$

$$=\underline{0{\cdot}187\text{ cm.}}$$

The work done $=$(average force) $\times$ (extension)

$$=\frac{10{,}000}{2}\times981\times0{\cdot}187\text{ ergs}$$

$$=\underline{918{,}700\text{ ergs.}}$$

52.

CONVERT the following temperatures to the Fahrenheit scale: 40° C., –5° C., –20° C.

$$40°\text{ C.}=40\text{ C. deg. above F.P.}=\left(\frac{9}{5}\times 40\right)\text{ F. deg. above F.P.}$$
$$=(32+72)°\text{ F.}$$
$$=\underline{104°\text{ F.}}$$

$$-5°\text{ C.}=5\text{ C. deg. below F.P.}=\left(\frac{9}{5}\times 5\right)\text{ F. deg. below F.P.}$$
$$=(32-9)°\text{ F.}$$
$$=\underline{23°\text{ F.}}$$

$$-20°\text{ C.}=32-\left(\frac{9}{5}\times 20\right)$$
$$=\underline{-4°\text{ F.}}$$

Convert the following temperatures to the Centigrade scale: 36° F., 9° F., –18° F.

$$36°\text{ F.}=(36-32)\text{ F. deg. above F.P.}$$
$$=\left(\frac{5}{9}\times 4\right)\text{ C. deg. above F.P.}$$
$$=\underline{2{\cdot}2°\text{ C.}}$$

$$9°\text{ F.}=\frac{5}{9}(9-32)°\text{ C.}=\underline{-12{\cdot}8°\text{ C.}}$$

$$-18°\text{ F.}=\frac{5}{9}(-18-32)°\text{ C.}=\underline{-27{\cdot}8°\text{ C.}}$$

53.

(*a*) What temperature has the same value on both the Centigrade and Fahrenheit scales?

(*b*) For what temperature is the Centigrade value twice the Fahrenheit value?

(*c*) For what temperature is the Fahrenheit value twice the Centigrade value?

Let the Fahrenheit and Centigrade readings by f and c respectively.

Then in each such problem we have to solve two simultaneous equations in f and c.

One of these equations is always the same and is the conversion equation used before:

$$(f-32)=\frac{9}{5}c$$

The other equation is an expression of the conditions stated in the particular problem.

$$(a)\quad \left.\begin{array}{l} c=f \\ \frac{9}{5}c=f-32 \end{array}\right\} \frac{9}{5}f=f-32$$

$$9f=5f-160$$

$$\therefore\quad \underline{\underline{f=-40^\circ \text{ F.}}}$$

$$\underline{\underline{c=-40^\circ \text{ C.}}}$$

$$(b)\quad \left.\begin{array}{l} c=2f \\ \frac{9}{5}c=f-32 \end{array}\right\} \frac{18}{5}f=5f-160$$

$$\therefore\quad \underline{\underline{f=-12{\cdot}3^\circ \text{ F.}}}$$

$$\underline{\underline{c=-24{\cdot}6^\circ \text{ C.}}}$$

$$(c)\quad \left.\begin{array}{l} 2c=f \\ \frac{9}{5}c=f-32 \end{array}\right\} \frac{9}{10}f=5f-160$$

$$\therefore\quad \underline{\underline{f=320^\circ \text{ F.}}}$$

$$\underline{\underline{c=160^\circ \text{ C.}}}$$

54.

A Centigrade thermometer reads 1·5° C. in melting ice and 99·4° C. in steam from water boiling at 76 cm. pressure. Assuming the bore to be uniform what is the correct temperature when the thermometer reads: (*a*) 20° C., (*b*) −10° C.?

Thermometer corrections should be dealt with in two stages: (1) Zero correction, (2) Fundamental Interval correction. The first stage is equivalent to shifting the scale bodily so that the zero is correct. (See diagram on next page.) This will obviously make equal corrections at all points of the scale. The second stage corrects for the divisions not being exact degrees. This part of the correction is proportional to the reading.

Zero error $=+1{\cdot}5°$

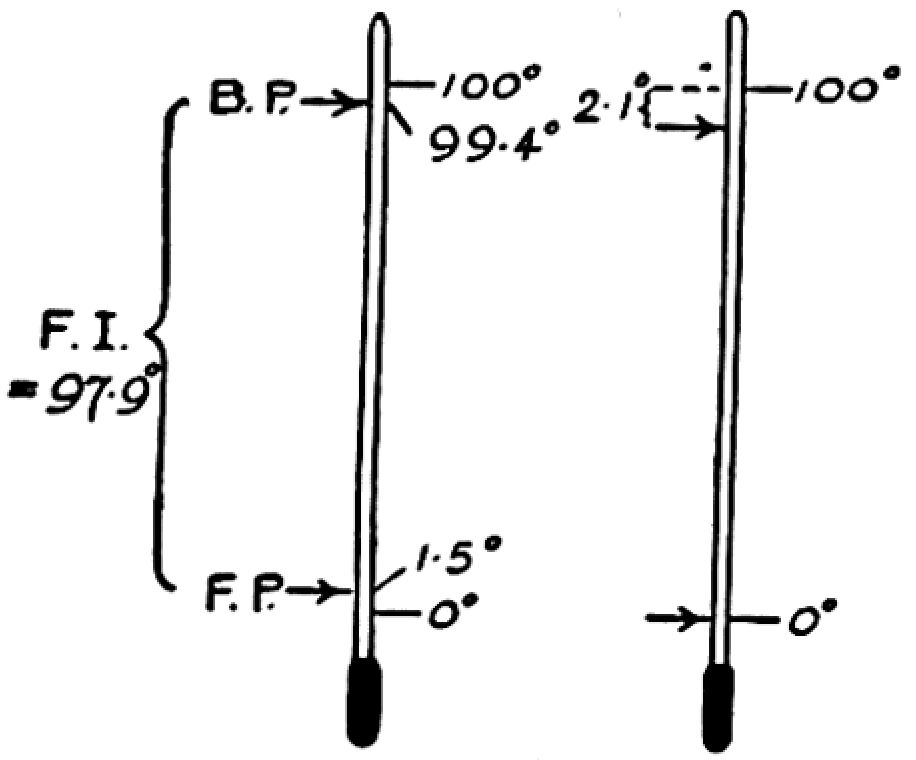
Zero correction $=-1{\cdot}5°$

Fundamental Interval $=(99{\cdot}4-1{\cdot}5)=97{\cdot}9$ divisions

Fundamental Interval error $=-2{\cdot}1°$

Fundamental Interval correction $=+2{\cdot}1°$ in $100°$

$=+0{\cdot}021°$ per $1°$



Stage (1)'

(*a*) For a reading of 20°, true temperature

$=20-1{\cdot}5+(20\times0{\cdot}021)$

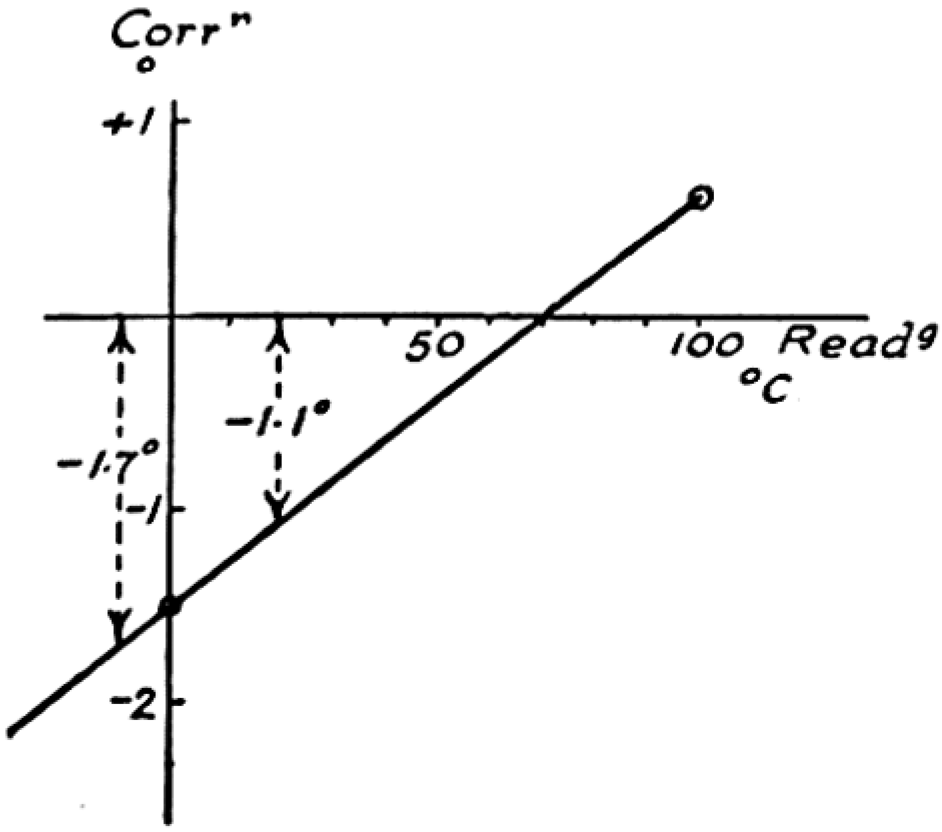
$=20-(1{\cdot}5-0{\cdot}42)$

$=\underline{18{\cdot}9°\text{ C.}}$

(*b*) For a reading of $-10°$, true temperature

$=-10-(1{\cdot}5+0{\cdot}21)$

$=\underline{-11{\cdot}7°\text{ C.}}$



This type of problem may be very conveniently solved graphically. The freezing-point and boiling-point corrections are plotted against their appropriate readings and the resulting two points are joined by a straight line, which provides a graph from which the total correction at any temperature may be read directly.

55.

By how much will a steel rail, 20 metres long, expand when heated from 10° C. to 15° C.?

Coefficient of linear expansion of steel$=0{\cdot}000012$ per deg. C.

Expansion of 1 metre of steel heated through 1° C.

$$=0{\cdot}000012 \text{ metre}$$

Expansion of 20 metres of steel heated through 5° C.

$$=(0{\cdot}000012\times5\times20) \text{ metre}$$
$$=0{\cdot}0012 \text{ metre}$$
$$=\underline{0{\cdot}12 \text{ cm.}}$$

56.

A glass rod and a steel rod are of equal length at 0° C. At 100° C. they differ in length by exactly 1 mm. What was their length at 0° C.?

The coefficients of expansion of glass and steel are respectively 8 and 12×20^{-6} per deg. C.

From the general expansion equation

$$L_t=L_o(1+at)$$

we have for the glass rod

$$_gL_t=L_o[1+(0{\cdot}000008\times100)]$$

and for the steel rod

$$_sL_t=L_o[1+(0{\cdot}000012\times100)]$$

and since

$$_sL_t-{}_gL_t=0{\cdot}1 \text{ cm.}$$
$$0{\cdot}1=L_o[(1+0{\cdot}0012)-(1+0{\cdot}0008)]$$
$$=0{\cdot}0004L_o$$
$$L_o=\frac{0{\cdot}1}{0{\cdot}0004} \text{ cm.}$$
$$=\underline{250 \text{ cm.}}$$

57.

Two rods of iron and brass respectively are made of equal length at a certain temperature. At 15° C. the brass rod is found to be shorter than the iron by 0·015 cm., the latter being 50·2 cm. long. What was the temperature at which the lengths were equal? The coefficients of linear expansion of iron and brass are 12 and 19×10^{-6} per degree respectively.

Since the brass rod is the shorter and brass has the greater coefficient of expansion, the required temperature must be above 15° C. Let it be $t°$ above.

For each rod: $L_{15+t}=L_{15}(1+at)$

$$\therefore \quad 50{\cdot}2(1+0{\cdot}000012t)=(50{\cdot}2-0{\cdot}015)(1+0{\cdot}000019t)$$
$$50{\cdot}2+0{\cdot}000602t=50{\cdot}185+0{\cdot}000954t$$
$$0{\cdot}015=t(0{\cdot}000954-0{\cdot}000602)$$
$$t=42{\cdot}7$$

$\therefore$ Temperature at which rods were of equal length

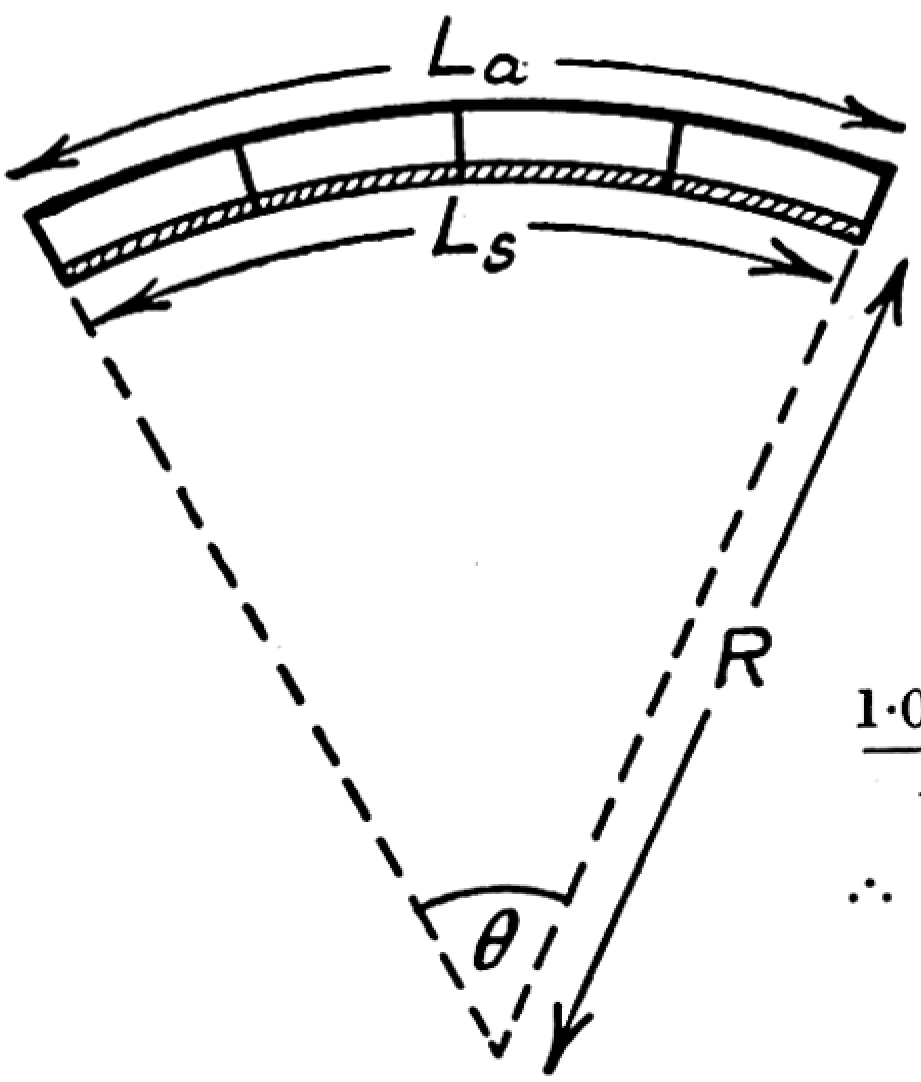
$=\underline{57{\cdot}7^\circ \text{ C.}}$

58.

Two straight thin bars, one of aluminium and the other of steel, are joined together at 0° C. side by side by short steel cross-pieces 1 cm. long, the centre lines of the bars being 1 cm. apart. When heated to 100° C., the composite bar becomes bent into the arc of a circle. Calculate the radius of this circle, taking the coefficients of expansion of aluminium and steel as 25 and 11×10^{-6} per deg. C. respectively.

At 100° C., owing to expansion of the cross-pieces, the distance between the centres of the bars becomes 1·0011 cm.

Referring to the diagram:



$$L_s=\theta R$$
$$L_a=\theta(R+1{\cdot}0011)$$
$$\frac{L_a}{L_s}=\frac{R+1{\cdot}0011}{R}=1+\frac{1{\cdot}0011}{R}$$

also

$$\frac{L_a}{L_s}=\frac{L_o[1+(100\times0{\cdot}000025)]}{L_o[1+(100\times0{\cdot}000011)]}$$
$$=\frac{1{\cdot}0025}{1{\cdot}0011}$$
$$\therefore \quad 1+\frac{1{\cdot}0011}{R}=\frac{1{\cdot}0025}{1{\cdot}0011}$$
$$\frac{1{\cdot}0011}{R}=\frac{1{\cdot}0025-1{\cdot}0011}{1{\cdot}0011}=\frac{0{\cdot}0014}{1{\cdot}0011}$$
$$\therefore \quad R=\frac{(1{\cdot}0011)^2}{0{\cdot}0014}$$
$$=\frac{1{\cdot}0022}{0{\cdot}0014}$$
$$=\underline{716 \text{ cm.}}$$

59.

Two clocks, having iron and brass pendulums respectively, have the same rate at 5° C. By how much per day will they differ at 25° C.?

Coefficient of linear expansion of iron $=12\times10^{-6}$ per deg. C.
Coefficient of linear expansion of brass $=18\times10^{-6}$ per deg. C.

For a simple pendulum of length L, the periodic time is given by:

$$T=2\pi\sqrt{\frac{L}{g}}$$

Assuming that this expression holds for an actual clock pendulum which only approximates to a "simple" pendulum, we may write, therefore:

$$\frac{T_b^2}{T_i^2}=\frac{L_b}{L_i}=\frac{1+(20\times0{\cdot}000018)}{1+(20\times0{\cdot}000012)}$$
$$=1+20(0{\cdot}000018-0{\cdot}000012)\text{ approx.*}$$
$$=1{\cdot}00012$$

$$\frac{T_b}{T_i}=\sqrt{1{\cdot}00012}=1{\cdot}00006\text{ approx.*}$$

Subtracting 1 from each side:

$$\frac{T_b-T_i}{T_i}=0{\cdot}00006$$

So the fractional difference between the periods of the two pendulums is 0·00006—that is, the clocks will differ by 0·00006 sec. per sec. and by 0·00006 days per day, the iron pendulum having the faster rate.

The difference per day is therefore equal to:

$$0{\cdot}00006\times24\times3600\text{ sec.}$$
$$=\underline{5{\cdot}18\text{ sec.}}$$

60.

A cylindrical steel rod, 4 cm. in diameter and 80 cm. long at 20° C., is heated to 100° C. and is prevented from expanding

* See p. 12.

by fixed contact pieces at the ends, exactly 80 cm. apart. Find the reactions at the ends at 100° C. if the coefficient of linear expansion of steel is 0·000011 per deg. C. and Young's modulus is 2×10^{12} dynes per sq. cm.

If free from restraint, expansion of rod

$$=80 \times 80 \times 0{\cdot}000011 \text{ cm.}$$
$$=0{\cdot}0704 \text{ cm.}$$

The required reaction equals the force required to shorten the rod by this amount. Let force$=F$ dynes.

Then

$$2 \times 10^{12} = \frac{\text{stress in dyne/cm.}^2}{\text{strain in cm./cm.}}$$
$$= \frac{F}{\pi \times 2^2} \times \frac{80}{0{\cdot}0704}$$

$$\therefore \qquad F = \frac{4 \times 3{\cdot}142 \times 0{\cdot}0704 \times 2 \times 10^{12}}{80} \text{ dynes}$$
$$= \frac{3{\cdot}142 \times 0{\cdot}0704 \times 10^8}{981} \text{ kgm. wt.}$$
$$= \underline{\underline{22{,}550 \text{ kgm. wt.}}}$$

61.

It is desired to use glass capillary tubing of bore 0·2 mm. in the construction of a Centigrade mercury thermometer reading from −10° C. to 100° C., in which the degree divisions shall be 2 mm. apart. What must be the capacity of the bulb?

The coefficients of cubical expansion of mercury and glass are 0·000182 and 0·000025 per deg. C. respectively.

Assuming that the −10° division is to be at the junction of the bulb and the stem, we may say that, if V is the volume of the bulb and stem up to the zero mark, the expansion of V c.c. of mercury heated from 0° to 1° must equal the volume of 2 mm. of the stem, remembering that we must use the coefficient of apparent expansion of mercury in glass.

$$\therefore \quad V \times (0{\cdot}000182 - 0{\cdot}000025) = \pi \times (0{\cdot}01)^2 \times 0{\cdot}2$$

$$V = \frac{3{\cdot}142 \times 0{\cdot}0001 \times 0{\cdot}2}{0{\cdot}000157}$$

$$=\frac{0.00006284}{0.000157}$$
$$=0.400 \text{ c.c.}$$

The volume of the bulb must therefore be this, minus the volume of the stem between zero and $-10°$,

$$=0.400-0.000628$$
$$=\underline{0.399 \text{ c.c.}}$$

62.

If the density of mercury at 0° C. is 13·596 gm. per c.c., what is its value at 60° C. if the coefficient of expansion of mercury is 0·000182 per deg. C.?

1 c.c. of a liquid at 0° C. expands to $(1+\alpha t)$ c.c. at $t°$ C. if α is the coefficient of expansion.

Therefore if ρ_0 is the density at 0° C. and ρ_t the density at $t°$ C.,

$$\rho_t=\frac{\rho_0}{1+\alpha t}$$

It is here sufficiently accurate to use the approximation

$$\rho_t=\rho_0(1-\alpha t)^*$$

Therefore for mercury:

$$\rho_{60}=13.596[1-(60\times 0.000182)]$$
$$=13.596(1-0.01092)$$
$$=13.596-0.149$$
$$=\underline{13.447 \text{ gm. per c.c.}}$$

[The correct value of 13·449 is given by using

$$\rho_{60}=\frac{13.596}{1+0.01092}$$

but the error of 2 in the fifth significant figure is small enough to warrant the choice of the simpler arithmetic on most occasions.]

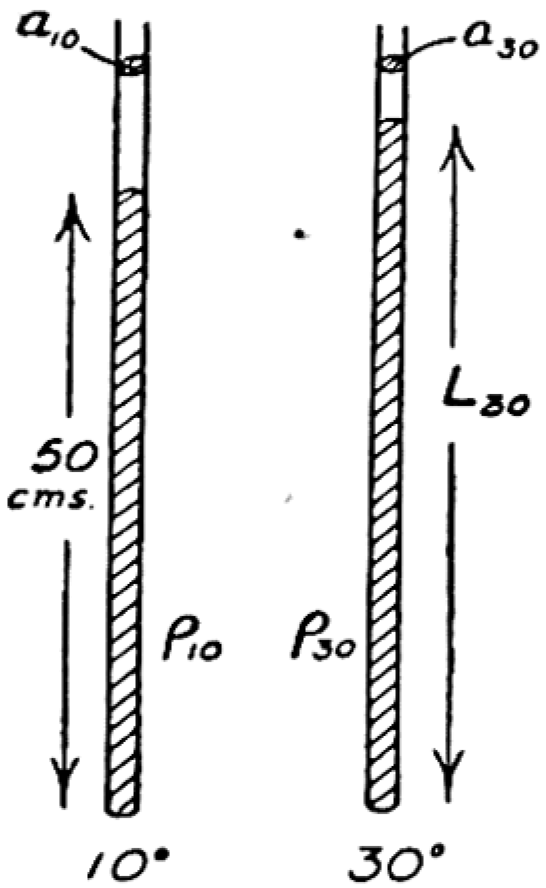
63.

A vertical glass tube, closed at the bottom, contains mercury to a height of 50 cm. at a temperature of 10° C. Find the height of the mercury column at 30° C.

Coefficient of cubical expansion of mercury
=0·000180 per deg. C.

Coefficient of linear expansion of glass =0·000008 per deg. C.

* See p. 12.



The mass of mercury in the tube is equal to the product of the volume and the density.

At 10° this equals $v_{10}\times\rho_{10}$
and at 30° $v_{30}\times\rho_{30}$

and, since the mass is constant,

$$v_{10}\times\rho_{10}=v_{30}\times\rho_{30}$$

The volume is the product of the length of column and the cross-sectional area of the tube.

So $$v_{10}=50\times a_{10}$$
$$v_{30}=L_{30}\times a_{30}$$

and, substituting these values in the first equation, we have:

$$50\times a_{10}\times\rho_{10}=L_{30}\times a_{30}\times\rho_{30}$$

whence $$L_{30}=50\times\frac{a_{10}}{a_{30}}\times\frac{\rho_{10}}{\rho_{30}}$$

The change in a involves the coefficient of superficial expansion of glass, which is twice the coefficient of linear expansion.

$\therefore$ $$a_{30}=a_{10}[1+(20\times2\times0{\cdot}000008)]$$

$$\frac{a_{10}}{a_{30}}=\frac{1}{1{\cdot}00032}$$

also $$\rho_{10}=\rho_{30}[1+(20\times0{\cdot}000180)] \text{ (see Ex. 56)}$$

$$\frac{\rho_{10}}{\rho_{30}}=1{\cdot}0036$$

$\therefore$ $$L_{30}=50\times\frac{1{\cdot}0036}{1{\cdot}00032}$$
$$=50\times1{\cdot}0036(1-0{\cdot}00032) \text{ approx.*}$$
$$=50(1{\cdot}0036-0{\cdot}00032) \text{ approx.}$$
$$=50{\cdot}164$$

At 30° C. the length of the mercury column is 50·16 cm.

64.

A barometer having a brass scale reads 77·24 cm. at a temperature of 20° C. What would be the reading at 0° C.?

* See p. 12.

Coefficient of cubical expansion of mercury
$=0{\cdot}00018$ per deg. C.
Coefficient of linear expansion of brass $=0{\cdot}000019$ per deg. C.

Consider the two factors affecting the correction:
(1) Cooling the scale causes it to contract and so increases the reading.
(2) Cooling the mercury increases its density and so decreases the reading.
(1) *Very nearly* the increase in reading due to scale contraction equals the contraction of 77·24 cm. of brass when cooled through 20° C. This equals $(77{\cdot}24\times20\times0{\cdot}000019)$.

$$\therefore \qquad H=77{\cdot}24[1+(20\times0{\cdot}000019)]$$

(2) Calling the final corrected height H_0, it is obvious that

$$H_0\rho_0=H\rho_{20}$$

to give equal pressures, and since

$$\rho_0=\rho_{20}[1+(20\times0{\cdot}00018)]$$

$$\begin{aligned} H_0&=\frac{H}{[1+(20\times0{\cdot}00018)]}\\ &=H[1-(20\times0{\cdot}00018)] \text{ approx.*}\\ &=77{\cdot}24[1+0{\cdot}00038][1-(20\times0{\cdot}00018)]\\ &=77{\cdot}24[1+0{\cdot}00038-0{\cdot}0036] \text{ approx.}\\ &=77{\cdot}24-0{\cdot}25\\ &=\underline{76{\cdot}99 \text{ cm.}}\end{aligned}$$

65.

A piece of metal weighs 40 gm. in air, 35·200 gm. when immersed in a liquid at 5° C. and 35·250 gm. at 35° C. Find the coefficient of expansion of the liquid if the coefficient of linear expansion of the metal is 0·0002 per deg. C.

Loss in weight at 5° C. $=4{\cdot}80$ gm.
Loss in weight at 35° C. $=4{\cdot}75$ gm.

By Archimedes' Principle:
Loss in Weight=(Volume of metal)(Density of liquid)

$$\therefore \qquad 4{\cdot}80=V_5\times\rho_5 \text{ and } 4{\cdot}75=V_{35}\times\rho_{35}$$

$$\begin{aligned}\frac{\rho_5}{\rho_{35}}&=\frac{4{\cdot}80}{4{\cdot}75}\cdot\frac{V_{35}}{V_5}\\ &=\frac{4{\cdot}80}{4{\cdot}75}[1+(30\times0{\cdot}00006)]\dagger\end{aligned}$$

* See p. 12.

† Coefficient of cubical expansion $=3\times$ Coefficient of linear expansion.

$$=\frac{4\cdot80}{4\cdot75}\times1\cdot0018$$

But since density is inversely proportional to the volume of a given mass:

$$\frac{\rho_5}{\rho_{35}}=1+30a$$

$$\therefore \qquad a=\frac{(4\cdot80\times1\cdot0018)-4\cdot75}{4\cdot75\times30}$$

$$=\underline{0\cdot00041 \text{ per deg. C.}}$$

Note that the same result would be obtained by calculating the coefficient of apparent expansion of the liquid and adding the coefficient of expansion of the solid, thus:

$$a=\frac{4\cdot80-4\cdot75}{4\cdot75\times30}+0\cdot00006$$

$$=0\cdot00041$$

66.

What will be the final temperature when 7 lb. of water at 150° F. are added to 10 lb. of water at 55° F. and thoroughly stirred?

Neglecting any heat lost to or gained from external bodies, we may write:

Heat lost by hot water=Heat gained by cold water

If the final temperature is t° F., then

$$7\times(150-t)=10\times(t-55) \text{ [in B.Th.U.]}$$

$$1050-7t=10t-550$$

$$1600=17t$$

$$t=94\cdot1$$

<u>The final temperature of the water will be 94·1° F.</u>

67.

How many calories are equivalent to a British Thermal Unit? (453·6 gm.=1 lb.)

$$1 \text{ B.Th.U.}=1 \text{ lb. deg. F.}$$

$$=453\cdot6 \text{ gm. deg. F.}$$

$$=453\cdot6\times\frac{5}{9} \text{ gm. deg. C.}$$

$$=453\cdot6\times\frac{5}{9} \text{ cal.}$$

$$=\underline{252\cdot0 \text{ cal.}}$$

68.

A piece of metal of mass 124·3 gm. is heated to 100° C. and dropped into a copper calorimeter of mass 45 gm. containing 200 gm. of water at 18° C. After stirring, the temperature of the water reaches a maximum value of 25·2° C. Neglecting heat loss from the calorimeter, calculate the specific heat of the metal. (Specific heat of copper=0·10.)

Heat lost by metal in cooling from 100° C. to 25·2° C. = Heat gained by calorimeter and water in being warmed from 18° C. to 25·2° C.

$$\therefore \quad 124{\cdot}3 \times s \times (100-25{\cdot}2) = [45 \times 0{\cdot}1 \times (25{\cdot}2-18)] + [200 \times (25{\cdot}2-18)]$$

$$124{\cdot}3 \times s \times 74{\cdot}8 = (4{\cdot}5+200) \times 7{\cdot}2$$

$$s = \frac{204{\cdot}5 \times 7{\cdot}2}{124{\cdot}3 \times 74{\cdot}8}$$

$$= \underline{0{\cdot}158}$$

69.

What volumes of iron, lead and aluminium have the same thermal capacity as a litre of water?

The specific heats of iron, lead and aluminium are respectively 0·12, 0·031 and 0·22; and their specific gravities, 7·5, 11·4 and 2·7.

The thermal capacity of a body is the quantity of heat required to raise its temperature by 1°; it is therefore equal to the product of the mass and the specific heat.

For 1000 c.c. (or 1000 gm.) of water, the thermal capacity =1000 cal.

The product of the mass and the specific heat has to be 1000 for each metal, that is

$$ms = 1000$$

or

$$m = \frac{1000}{s}$$

The volume is given by dividing the mass by the density:

$$v = \frac{m}{\rho} = \frac{1000}{\rho s}$$

So the required volumes are:

for iron $$v = \frac{1000}{7{\cdot}5 \times 0{\cdot}12} = \underline{1111 \text{ c.c.}}$$

for lead $$v = \frac{1000}{11{\cdot}4 \times 0{\cdot}031} = \underline{2830 \text{ c.c.}}$$

for aluminium $v=\frac{1000}{2{\cdot}7\times0{\cdot}22}=\underline{1683 \text{ c.c.}}$

70.

Two exactly similar copper calorimeters, each of mass 50 gm., contain 100 c.c. of water and turpentine respectively at a temperature of 90° C. They are allowed to cool under identical conditions and it is found that the times for cooling from 70° to 60° C. are 5 min. 20 sec. for the water and 2 min. 7 sec. for the turpentine. What is the specific heat of turpentine?

Density of turpentine=0·87 gm. per c.c.
Specific heat of copper=0·10

The rate of loss of heat from a surface depends on the degree of polish of the surface, its area, and its temperature excess with respect to its surroundings. As the two calorimeters are identical as regards surface polish and temperature (70° to 60°), and as they contain equal volumes of hot liquid and therefore their heated areas are equal, the rates of loss of heat must be equal.

For the calorimeter containing 100 gm. of water:

$$\text{Loss of heat in 320 sec.}=[(50\times0{\cdot}1)+100]\times(70-60)\text{ cal.}$$

$$\text{Rate of loss of heat}=\frac{(5+100)\times10}{320}\text{ cal. per sec.}$$

For the calorimeter containing ($100\times0{\cdot}87$) gm. of turpentine:

$$\text{Loss of heat in 127 sec.}=[(50\times0{\cdot}1)+87s]\times(70-60)\text{ cal.}$$

(where s is the specific heat of turpentine)

$$\text{Rate of loss of heat}=\frac{(5+87s)\times10}{127}\text{ cal. per sec.}$$

$$\therefore \quad \frac{105\times10}{320}=\frac{(5+87s)\times10}{127}$$

$$320(5+87s)=127\times105$$

$$\therefore \quad s=\left(\frac{127\times105}{320}-5\right)\div87$$

$$=\underline{0{\cdot}42}$$

71.

A copper calorimeter of water-equivalent 15 gm., containing 50 c.c. of hot water, is placed in an enclosure whose walls are

maintained at constant temperature, and it is found that its temperature falls from 70° C. to 60° C. in 2 min. 45 sec. The water is replaced by an equal volume of hot paraffin oil of specific gravity 0·80 and specific heat 0·53. What time will now be taken to cool through the same temperature range, 70° to 60°?

Let T be the required time.

Rate of loss of heat of calorimeter and water

$$=\frac{\text{Heat lost in cooling from } 70° \text{ to } 60°}{\text{Time taken to cool from } 70° \text{ to } 60°}$$

$$=\frac{(\text{Water-equivalent of cal.}+\text{mass of water})\times(70-60)}{165}$$

$$=\frac{(15+50)\times 10}{165} \text{ cal. per sec.}$$

Similarly, rate of loss of heat of calorimeter and oil

$$=\frac{[15+(50\times 0{\cdot}8\times 0{\cdot}53)]\times 10}{T} \text{ cal. per sec.}$$

Since equal volumes of the two liquids are used, the same area of the calorimeter surface is emitting heat through the same temperature range, and therefore the rates of loss of heat are the same in the two cases.

$$\therefore \qquad \frac{(15+50)\times 10}{165}=\frac{[15+(50\times 0{\cdot}8\times 0{\cdot}53)]\times 10}{T}$$

$$T=\frac{165\times(15+21{\cdot}2)}{65}$$

$$=92 \text{ sec.}$$

The required time is 1 min. 32 sec.

72.

A can of hot water is stood on a table to cool. Its temperature is observed to fall from 60° to 50° C. in 10 min. 30 sec. Assuming Newton's Law of cooling to hold, how long will it take for the water to cool (*a*) from 50° to 40°, and (*b*) from 40° to 30°, if the mean temperature of the surroundings is 15° C.?

According to Newton's Law, the rate of cooling is proportional to the excess temperature of the hot body with respect to its surroundings.

From 60° to 50°:

Mean temperature	$=55°$
Mean excess temperature	$=(55°-15°)$
	$=40°$

From 50° to 40°:

Mean temperature	$=45°$
Mean excess temperature	$=(45°-15°)$
	$=30°$

By Newton's Law:

$$\frac{\text{Rate of cooling from } 60° \text{ to } 50°}{\text{Rate of cooling from } 50° \text{ to } 40°}=\frac{40}{30}$$

The rate of cooling over a period T sec. is

$$\left(\frac{\text{Fall of temperature in } T \text{ sec.}}{T}\right)$$

(*a*) $\therefore$ If T_a is the time taken to cool from 50° to 40°,

$$\frac{\left(\frac{60-50}{630}\right)}{\left(\frac{50-40}{T_a}\right)}=\frac{40}{30}$$

$$\frac{10}{630}\cdot\frac{T_a}{10}=\frac{4}{3}$$

$$T_a=840 \text{ sec.}$$
$$=\underline{14 \text{ min.}}$$

(*b*) Similarly, if T_b is the time to cool from 40° to 30°,

$$\frac{\left(\frac{60-50}{630}\right)}{\left(\frac{40-30}{T_b}\right)}=\frac{40}{20}$$

$$T_b=1260 \text{ sec.}$$
$$=\underline{21 \text{ min.}}$$

73.

If the latent heat of steam is 540 cal. per gm., what is its value in B.Th.U. per lb.?

Let there be M gm. to a lb.

Then the latent heat of steam $=540\times M$ cal. per lb.

$$=540\times M \text{ gm. deg. C. per lb.}$$

$$=\frac{540\times M}{M} \text{ lb. deg. C. per lb.}$$

$$=\frac{540\times M}{M\times\frac{5}{9}} \text{ lb. deg. F. per lb.}$$

$$=540\times\frac{9}{5} \text{ lb. deg. F. per lb.}$$

$$=972 \text{ lb. deg. F. per lb.}$$

$$=\underline{972 \text{ B.Th.U. per lb.}}$$

74.

150 gm. of water are introduced into a calorimeter of mass 53 gm. and the temperature is observed to be 25° C. Dry ice is then carefully dropped into the water until the temperature has fallen to 15° C. The total mass of the calorimeter and contents is found finally to be 219·3 gm. What is the water equivalent of the calorimeter? (Latent heat of fusion of ice=80 cal. per gm.)

Mass of ice introduced $=219{\cdot}3-150-53=16{\cdot}3$ gm.

Let x be the water equivalent of the calorimeter.

Heat gained by ice in melting and in being warmed to 15° C. = Heat lost by calorimeter and water in cooling from 25° C. to 15° C.

$$(16{\cdot}3\times 80)+(16{\cdot}3\times 15)=(x\times 10)+(150\times 10)$$

$$16{\cdot}3\times 95=10x+1500$$

$$x=\frac{(16{\cdot}3\times 95)-1500}{10}$$

$$=\underline{4{\cdot}85 \text{ gm.}}$$

75.

Steam at 100° C. is passed into a calorimeter of mass 55 gm., containing 150 gm. of a mixture of ice and water, until all the ice is melted. The mass of calorimeter and contents is then found to be 217·5 gm. How much ice was originally present in the mixture?

Latent heat of fusion of ice $=80$ cal. per gm.
Latent heat of steam $=540$ cal. per gm.

So long as any ice remains, the temperature of the mixture will be 0° C. The temperature therefore is constant throughout the experiment and there is no need to consider the water equivalent of the calorimeter.

The mass of steam condensed is equal to

$$[217{\cdot}5-(55+150)] \text{ gm.} = 12{\cdot}5 \text{ gm.}$$

Let the mass of ice present be x gm.

Then: Heat lost by steam in condensing and being cooled to 0° C. = Heat gained by ice in being melted

$$(12{\cdot}5\times540)+(12{\cdot}5\times100)=80\times x$$

$$\therefore \qquad x=\frac{640\times12{\cdot}5}{80}$$

$$=100$$

The mixture of ice and water contained 100 gm. of ice.

76.

A piece of metal of mass 1·2 gm. is heated to 100° C. and dropped into a Bunsen ice calorimeter, causing the mercury thread to recede a distance of 3 cm. in the capillary tube of 1 sq. mm. cross-section. Calculate the specific heat of the metal, taking the densities of ice and water at 0° C. as 0·917 and 1·000 gm./c.c. respectively. (Latent heat of fusion of ice $=80$ cal./gm.)

Contraction of mixture of ice and water caused by the melting of some of the ice

$$=(3\times0{\cdot}01) \text{ c.c.}$$
$$=0{\cdot}03 \text{ c.c.}$$

Contraction of 1 gm. of ice on melting

$$=(\text{Volume of 1 gm. of ice}) - (\text{Volume of 1 gm. of water})$$

$$=\left(\frac{1}{0{\cdot}917}-1\right) \text{ c.c.}$$

$$=\frac{0{\cdot}083}{0{\cdot}917} \text{ c.c.}$$

$$=0{\cdot}0905 \text{ c.c.}$$

Mass of ice melted $\qquad = \dfrac{0{\cdot}03}{0{\cdot}0905}$ gm.

Heat gained by ice $\qquad = \left(\dfrac{0{\cdot}03}{0{\cdot}0905} \times 80\right)$ cal.

This must equal the heat lost by the metal

$$\therefore \qquad 1{\cdot}2 \times s \times 100 = \frac{0{\cdot}03 \times 80}{0{\cdot}0905}$$

$$s = \frac{0{\cdot}03 \times 80}{0{\cdot}0905 \times 1{\cdot}2 \times 100}$$

$$= \underline{0{\cdot}22}$$

77.

In a continuous flow calorimeter, a heating current of 1·5 amp. causes a temperature rise of 10° C. with a flow of liquid of 15·74 gm. per minute, the potential difference across the heating coil being 6·0 volts. In a second experiment, the same rise of temperature is produced with a current of 2·0 amp., a flow of 34·35 gm. per minute and a P.D. of 8·0 volts. Calculate the specific heat of the liquid. $J=4{\cdot}18$ joules per calorie.

Since the temperatures are the same in the two experiments, it is reasonable to suppose that the same rate of loss of heat occurs. Let this be h calories per second. The quantity of heat available from IV watts is $\dfrac{IV}{J}$ cal. per sec.

$$\therefore \qquad \frac{1{\cdot}5 \times 6}{4{\cdot}18} = \frac{10 \times 15{\cdot}74 \times s}{60} + h$$

where s is the specific heat of the liquid.

and
$$\frac{2 \times 8}{4{\cdot}18} = \frac{10 \times 34{\cdot}35 \times s}{60} + h$$

subtracting,
$$\frac{16-9}{4{\cdot}18} = \frac{10 \times 18{\cdot}61 \times s}{60}$$

$$\underline{\underline{s = 0{\cdot}54 \text{ cal./gm./}^\circ \text{ C.}}}$$

78.

Water is boiled in a rectangular steel tank, the bottom of which is 5 mm. thick. If the water level falls steadily at the rate of 1 cm. in 5 minutes, what is the temperature of the lower surface of the tank?

Thermal conductivity of steel $=0\cdot12$ cal./cm./sec./deg. C.
Latent heat of steam $=540$ cal. per gm.

Let the area of the bottom plate of the tank be A sq. cm. and the temperature of the lower surface $\theta°$.

Since the temperature of the upper surface, in contact with the boiling water, may be assumed to be 100° C.,

$$\text{Temperature gradient in the steel plate} = \frac{\theta-100}{0\cdot5} \text{ deg. per cm.}$$

The heat passing by conduction through the plate is sufficient to vaporize A c.c. of water in 5 min., or A gm. in 5 min.

This requires $540A$ calories in 5 min.

$\therefore$ The rate of flow of heat through the plate is:

$$\frac{Q}{t}=\frac{540A}{5\times60} \text{ cal. per sec.}$$

Substituting in the conductivity equation

$$\frac{Q}{t}=\frac{kA(\theta_1-\theta_2)}{d}$$

we have

$$\frac{540\times A}{5\times60}=0\cdot12\times A\times\frac{(\theta-100)}{0\cdot5}$$

$$\theta-100=\frac{9}{1\cdot2}$$

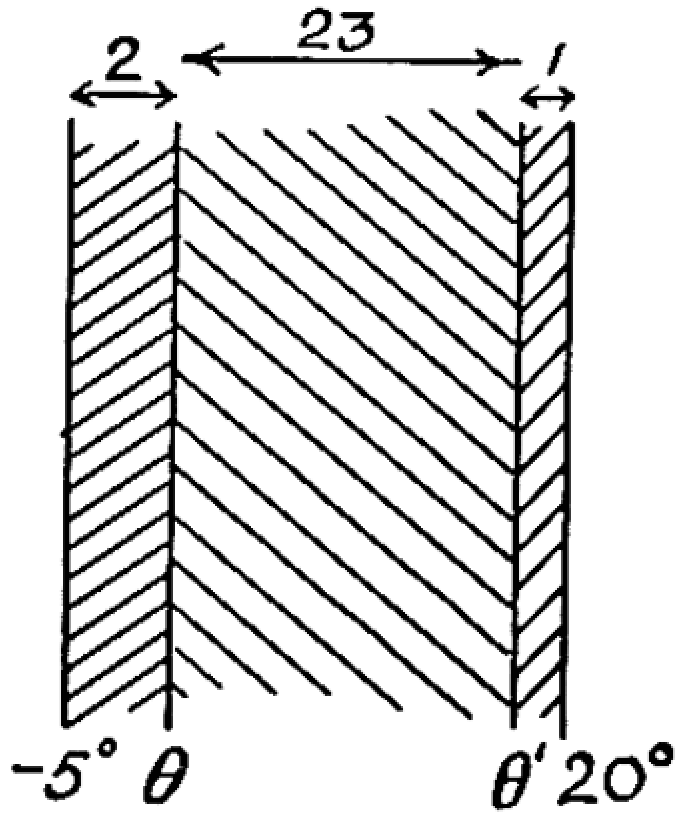
$$=7\cdot5$$

$$\underline{\theta=107\cdot5°\text{ C.}}$$

79.

The walls of a room are formed of parallel layers in contact of cement, brick and wood of thicknesses 2, 23 and 1 cm. respectively. Find how much heat passes by conduction through each square metre of wall per minute if the temperatures of the air in contact with the wall are $-5°$ C. outside and $20°$ C. inside. The coefficients of thermal conductivity for cement, brick and wood are $0\cdot0007$, $0\cdot006$ and $0\cdot0004$ c.g.s. units respectively.

It is necessary to realise that in a composite slab such as we have here, the rate of flow of heat is constant throughout the slab when conditions have become steady; otherwise there would be an accumulation or a diminution of heat at the interfaces. Considering the electrical analogy, we may say that the three layers are "in series" and therefore carry the same "current."



Applying the conductivity equation $\frac{Q}{t}=\frac{kA(\theta_1-\theta_2)}{d}$ to each of the layers, and calling the temperatures at the interfaces θ and θ',

$$\frac{Q}{t}=\frac{0{\cdot}0007\times10^4\times(\theta+5)}{2};\quad (\theta+5)=\frac{2Q}{7t}$$

$$\frac{Q}{t}=\frac{0{\cdot}006\times10^4\times(\theta'-\theta)}{23};\quad (\theta'-\theta)=\frac{23Q}{60t}$$

$$\frac{Q}{t}=\frac{0{\cdot}0004\times10^4\times(20-\theta')}{1};\quad (20-\theta')=\frac{Q}{4t}$$

Adding the three equations eliminates θ and θ':

$$25=\frac{Q}{t}\left(\frac{2}{7}+\frac{23}{60}+\frac{1}{4}\right)$$

$$=\frac{Q}{t}\cdot\frac{386}{420}$$

$$\therefore\qquad \frac{Q}{t}=27{\cdot}21 \text{ cal./sq. m./sec.}$$

Required rate of flow of heat$=$<u>1633 cal./sq. m./min.</u>

80.

Water flows at the rate of 440 gm. per minute through a glass tube 60 cm. long surrounded by dry steam at 100° C., the inlet and outlet temperatures of the water being 20° and 45° C. respectively. If the external diameter of the tube is 1 cm., what is its internal diameter? What is the minimum rate of flow of steam required to ensure that every part of the tube is surrounded by steam?

Thermal conductivity of glass$=0{\cdot}0015$ cal./sec./cm./deg. C.

In the conductivity equation

$$\frac{Q}{t}=\frac{kA(\theta_1-\theta_2)}{d}$$

$\frac{Q}{t}$ equals the rate of absorption of heat by the water

$$=\frac{440\times(45-20)}{60}\text{ cal. per sec.}$$

A is the mean of the area of the surface of the tube exposed to the steam and that in contact with the water

$$=\left[\frac{\pi(1+D)\times 60}{2}\right]\text{ sq. cm.}$$

D being the internal diameter.

$(\theta_1-\theta_2)=(100-20)°$ at one end of the tube and $(100-45)°$ at the other. Mean

$$=(100-32{\cdot}5)°=67{\cdot}5°$$

d is the thickness of the walls of the tube

$$=\frac{(1-D)}{2}\text{ cm.}$$

Substituting these values in the equation, we have:

$$\frac{440\times 25}{60}=0{\cdot}0015\times\frac{22}{7}\times\frac{(1+D)}{2}\times 60\times\frac{67{\cdot}5\times 2}{(1-D)}$$

$$\therefore \qquad \frac{(1+D)}{(1-D)}=\frac{175}{18{\cdot}23}$$

$$D=\underline{0{\cdot}81\text{ cm.}}$$

Since heat is being taken from the steam by the water at the rate of $\left(\frac{440\times 25}{60}\right)$ cal. per sec., and each gram of steam in condensing yields 540 cal., the mass of steam available must be in excess of $\left(\frac{440\times 25}{60\times 540}\right)$ gm. per sec. This is equal to ·339 gm. per sec.

81.

Find the value of the mechanical equivalent of heat in ft. lb. per B.Th.U. given its value in ergs per cal. as $4{\cdot}18\times 10^7$.

Let there be M gm. to a lb.

$$1 \text{ ft. lb.} = 12 \times 2{\cdot}54 \text{ cm. lb. } (1 \text{ in.} = 2{\cdot}54 \text{ cm.})$$
$$= 12 \times 2{\cdot}54 \times M \text{ cm. gm.}$$
$$= 12 \times 2{\cdot}54 \times M \times 981 \text{ cm. dynes or ergs}$$

$$1 \text{ B.Th.U.} = M \times \frac{5}{9} \text{ cal.}$$

So $$J = 4{\cdot}18 \times 10^7 \times M \times \frac{5}{9} \text{ ergs per B.Th.U.}$$

$$= \frac{4{\cdot}18 \times 10^7 \times M \times \frac{5}{9}}{12 \times 2{\cdot}54 \times M \times 981} \text{ ft. lb. per B.Th.U.}$$

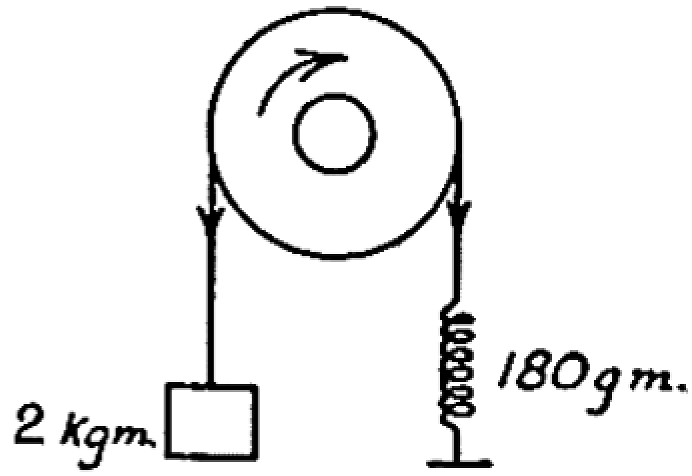
$$= \underline{779 \text{ ft. lb. per B.Th.U.}}$$

82.

An aluminium flywheel of radius 7·5 cm. and mass 1000 gm. is driven at the rate of 100 revolutions per minute. Its rim is braked by a silk band, one end of which carries a load of 2 kgm. while the other is attached to a spring balance reading 180 gm. Assuming all the heat generated is spent in raising the temperature of the flywheel, calculate the rise of temperature per minute.

Specific heat of aluminium $= 0{\cdot}22$.

$J = 4{\cdot}18 \times 10^7$ ergs per cal.



The difference in the tensions in the band on the two sides of the wheel is equal to

$$(2000 - 180) \times 981 \text{ dynes}$$

This is the frictional force acting on the rim of the wheel. The work done per revolution in moving the rim against this force is

$$(2\pi \times 7{\cdot}5 \times 1820 \times 981) \text{ ergs}$$

The work done per min. $= (100 \times 2\pi \times 7{\cdot}5 \times 1820 \times 981)$ ergs

Heat generated per min. $= \dfrac{(1500\pi \times 1820 \times 981)}{4{\cdot}18 \times 10^7}$ cals.

$$= \frac{(1{\cdot}5\pi \times 182 \times 0{\cdot}981)}{4{\cdot}18} \text{ cal.}$$

Rise of temperature per minute in wheel of thermal capacity $(1000 \times 0{\cdot}22)$ cals. per deg. C.

$$=\left(\frac{1{\cdot}5\pi \times 182 \times 0{\cdot}981}{4{\cdot}18 \times 220}\right)^{\circ} \text{C.} = \underline{0{\cdot}914^{\circ}\ \text{C.}}$$

83.

From what height must a piece of lead be dropped if the heat generated on impact with the ground is just sufficient to melt it? Assume that all the heat is confined to the lead.

Initial temperature of lead $=20^{\circ}$ C.
Melting point of lead $=327^{\circ}$ C.
Specific heat of lead $=0{\cdot}030$
Latent heat of fusion of lead $=5$ cal./gm.
$J=4{\cdot}18 \times 10^{7}$ ergs/cal.

Let the mass of lead be m gm.

Heat required to raise the temperature of m gm. of lead from 20° C. to 327° C. and to melt it

$$=(m \times 0{\cdot}03 \times 307)+(m \times 5) \text{ cal.}$$
$$=14{\cdot}21m \text{ cal.}$$

This is equivalent to $14{\cdot}21m \times 4{\cdot}18 \times 10^{7}$ ergs, which is therefore equal to the kinetic energy which the lead must possess just before impact. This kinetic energy is equal to the potential energy which the lead will have lost in falling.

Let the height from which the lead is dropped be h cm.

Then the loss in potential energy

$$=mgh=14{\cdot}21m \times 4{\cdot}18 \times 10^{7}$$

$$\therefore \qquad h=\frac{14{\cdot}21 \times 4{\cdot}18 \times 10^{7}}{981}$$

$$=6{\cdot}054 \times 10^{5} \text{ cm.} = \underline{6{\cdot}054 \text{ km.}}$$

84.

At its rated voltage, 100v., a 100-watt lamp filament attains a temperature of 2600° C., air temperature being 20° C. When the P.D. is reduced to 14v. the current through the filament is 0·4 amp. Use Stefan's Law to calculate the temperature of the filament, assuming that it radiates as a "black body" and neglecting loss of heat by convection and conduction.

Let the absolute temperature of the filament be T°.

Power used at this temperature $=14 \times 0{\cdot}4=5{\cdot}6$ watts.

By Stefan's Law, energy radiated is proportional to the

difference between the fourth powers of the absolute temperatures of the radiator and the surroundings.

$$\therefore \qquad \frac{60}{5 \cdot 6} = \frac{2873^4 - 293^4}{T^4 - 293^4}$$

$$T^4 - 293^4 = \frac{5 \cdot 6}{60}(2873^4 - 293^4)$$

$$= 0 \cdot 09333(6 \cdot 813 \times 10^{13} - 7 \cdot 37 \times 10^9)$$

$$= 0 \cdot 6356 \times 10^{13}$$

$$T^4 = 0 \cdot 6356 \times 10^{13} + 7 \cdot 37 \times 10^9$$

$$= 6 \cdot 363 \times 10^{12}$$

$$T = \sqrt[4]{6 \cdot 363} \times 10^3$$

$$\log T = \tfrac{1}{4} \log 6 \cdot 363 + 3$$

$$= 3 \cdot 2009$$

$$T = 1588^\circ \text{ A}$$

$$= \underline{1315^\circ \text{ C.}}$$

85.

A narrow, uniform glass tube, sealed at one end, contains a mercury pellet 5 cm. long. When the tube is held vertically with the sealed end up, the length of air column imprisoned by the mercury pellet is 25·6 cm. When the tube is inverted, the air column is 22·4 cm. long. What is the atmospheric pressure?

In each position, the pressure below the pellet exceeds that above by 5 cm. of mercury.

Therefore, calling the atmospheric pressure P, the pressure of the air in the tube is $(P-5)$ in the first position and $(P+5)$ in the second.

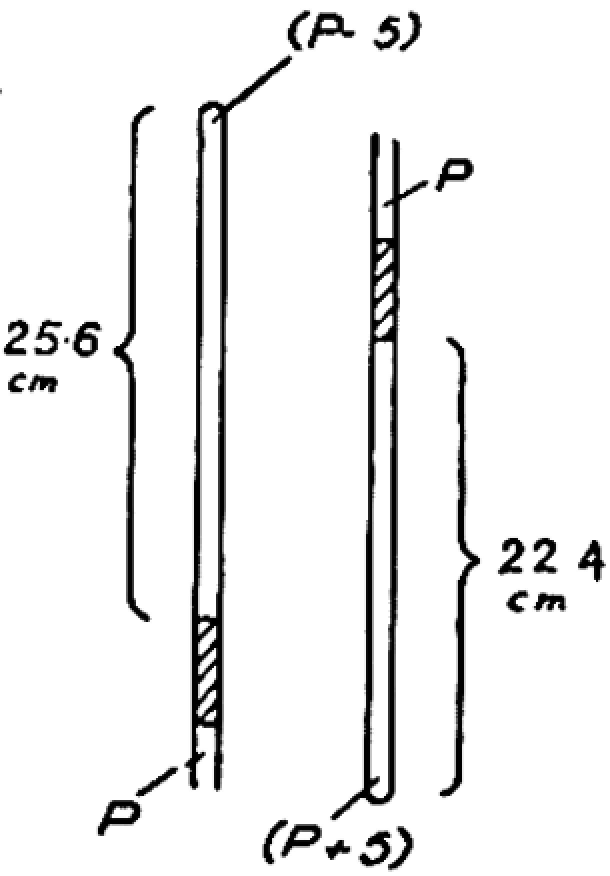
Applying Boyle's Law, $p_1v_1 = p_2v_2$, we have:

$$(P-5) \times 25 \cdot 6 = (P+5) \times 22 \cdot 4$$

since the volume of the air column is proportional to its length.

$$\therefore \qquad 3 \cdot 2P = 240$$

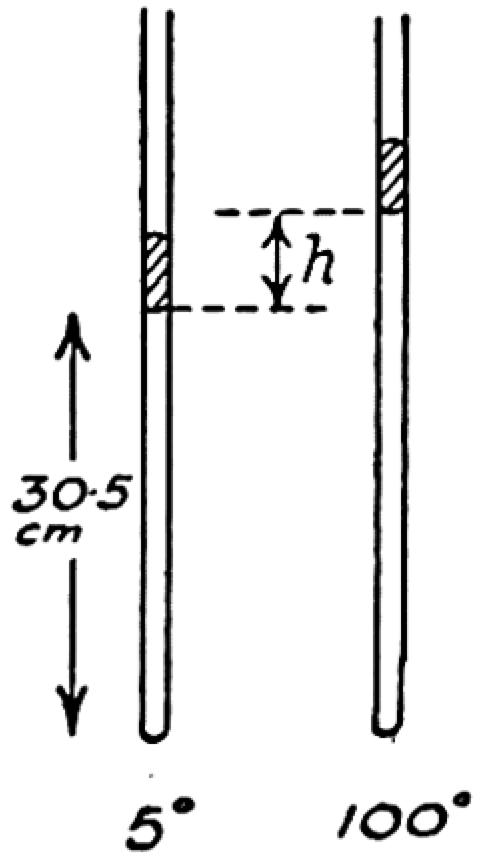
$$\underline{P = 75 \text{ cm. of mercury}}$$



86.

A uniform vertical glass tube, open at the top and closed at the bottom, contains air and a mercury pellet 3·0 cm. long, the lower

end of which is 30·5 cm. above the bottom of the tube when the temperature is 5° C. How far will the pellet rise if the tube is heated to 100° C.



The pressure on the air enclosed by the mercury pellet remains constant when the temperature is raised, being equal to atmospheric pressure plus 3 cm. of mercury.

Therefore the air expands at constant pressure and by Charles's Law:

$$v_5 = v_0\left(1+\frac{5}{273}\right)$$

$$v_{100} = v_0\left(1+\frac{100}{273}\right)$$

$$\therefore \qquad \frac{v_{100}}{v_5} = \frac{1+\frac{100}{273}}{1+\frac{5}{273}}$$

$$= \frac{273+100}{273+5}$$

$$= \frac{373}{278}$$

(the volumes are proportional to the absolute temperatures).

Suppose the pellet rises h cm. Then, the tube being uniform, the volume of air is proportional to the length of air column, and

$$\frac{30{\cdot}5+h}{30{\cdot}5} = \frac{373}{278}$$

$$1+\frac{h}{30{\cdot}5} = 1+\frac{95}{278}$$

$$\therefore \qquad h = 30{\cdot}5 \times \frac{95}{278}$$

$$= 10{\cdot}4 \text{ cm.}$$

The pellet will rise 10·4 cm.

87.

A collapsed balloon contains 150 cub. ft. of air at atmospheric pressure. It is filled with hydrogen from 20 cylinders, each holding 8 cub. ft. of the gas at 100 atmospheres pressure, the total

capacity of the envelope being 15,000 cub. ft. when fully distended. What pressure is exerted by the mixed gases in the balloon?

By Dalton's law of partial pressures, the pressure of the mixed gases is equal to the sum of the pressures which the air and the hydrogen would exert if each occupied 15,000 cub. ft. Call these pressures p_a and p_h respectively.

By Boyle's Law, $p_1v_1=p_2v_2$

For the air: $1\times 150=p_a\times 15{,}000$

$$p_a=0{\cdot}01 \text{ atm.}$$

For the hydrogen:

$$100\times(20\times 8)=p_h\times 15{,}000$$

$$p_h=\frac{100\times 160}{15000}$$

$$=1{\cdot}067 \text{ atm.}$$

Total pressure $=p_a+p_h$

$$=\underline{1{\cdot}077 \text{ atm.}}$$

88.

What mass of air will be expelled from a cubical room 4 metres high, when it is heated from 15° C. to 20° C., the pressure remaining constant at 78 cm. of mercury? The density of air at N.T.P. is 1·29 gm. per litre.

Since Mass $(M)=$volume $(V)\times$density (ρ)

$$M_{20}-M_{15}=V(\rho_{20}-\rho_{15})=\text{mass expelled}$$

Density at 78 cm. and 15° C. $=\left(1{\cdot}29\times\frac{78}{76}\times\frac{273}{288}\right)$ gm. per litre

Density at 78 cm. and 20° C. $=\left(1{\cdot}29\times\frac{78}{76}\times\frac{273}{293}\right)$ gm. per litre

Volume of room $=(40)^3=64{,}000$ litres

Mass expelled $=\left(\frac{64000\times 1{\cdot}29\times 78\times 273}{76}\right)\left(\frac{1}{288}-\frac{1}{293}\right)$

$$=\frac{64000\times 1{\cdot}29\times 78\times 273\times 5}{76\times 288\times 293}$$

$$=\underline{1371 \text{ gm.}}$$

89.

A constant volume air thermometer is used to measure the temperature of a water bath. With the bulb in ice, the mercury surface in the reservoir is 21·6 cm. above that at the constant

volume mark, and with the bulb in the water bath, the level in the reservoir has to be raised a further 13·2 cm. to keep the volume constant. The height of the barometer being 75·6 cm., what is the temperature of the water bath?

Since no steam reading is given, we must use the relation that, at constant volume, pressure is proportional to absolute temperature.

Total pressure of air in bulb at 0° C.

$$=75{\cdot}6+21{\cdot}6=97{\cdot}2 \text{ cm.}$$

Total pressure at the temperature of the water bath, say $t°$ C.

$$=75{\cdot}6+21{\cdot}6+13{\cdot}2=110{\cdot}4 \text{ cm.}$$

$$\therefore \qquad \frac{97{\cdot}2}{110{\cdot}4}=\frac{273}{273+t}$$

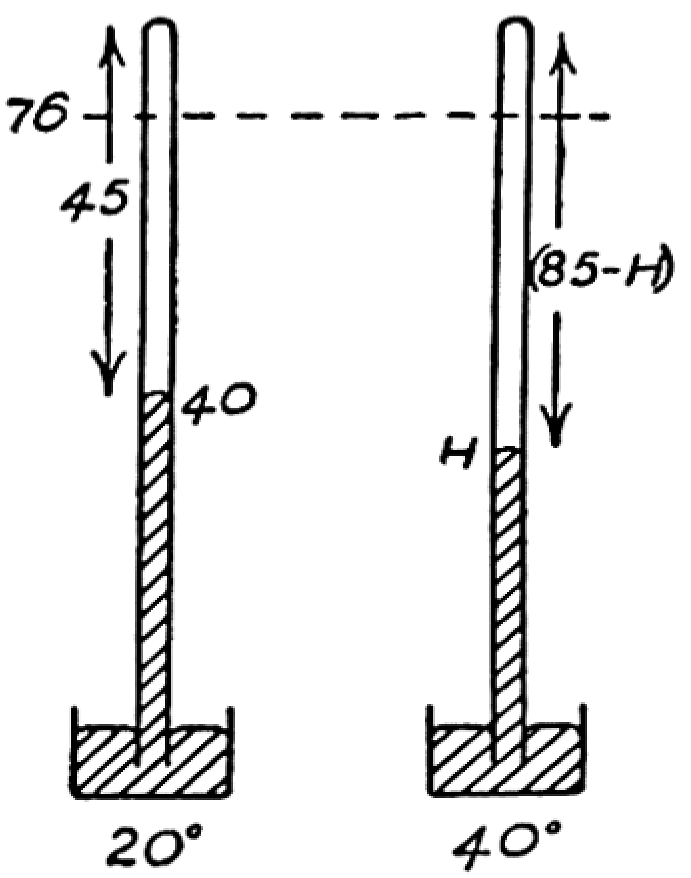
$$273+t=273\times\frac{110{\cdot}4}{97{\cdot}2}$$

$$=310{\cdot}1$$

$$t=\underline{37{\cdot}1° \text{ C.}}$$

90.

Air is introduced into the space above the mercury in a barometer tube until the mercury stands at 40 cm. when the atmospheric pressure is 76 cm. and the temperature 20° C., the length of the air column being 45 cm. What would be the height of the mercury if the temperature were raised to 40° C., assuming the tube to be of uniform bore and neglecting the expansion of the mercury?



Any change in the mercury level will affect both the pressure and the volume of the air in the tube, and so we are concerned with a change in temperature, pressure and volume.

The general gas equation is therefore required:

$$PV=RT$$

or $$\frac{P_1V_1}{T_1}=\frac{P_2V_2}{T_2}$$

We have $$\left\{\begin{array}{l} T_1=273+20=293° \\ T_2=273+40=313° \\ P_1=76-40=36 \text{ cm.} \\ V_1=45a \end{array}\right.$$

where a is the cross-sectional area of the tube. There are two unknowns, P_2 and V_2; but they are not independent of each other, for if the height of the mercury column is H,

$$V_2=a(85-H)$$

the height of the top of the tube being 85 cm.

and
$$P_2=76-H$$

$\therefore$ Substituting in the gas equation:

$$\frac{36\times 45a}{293}=\frac{(76-H)\times(85-H)a}{313}$$

$$H^2-161H+6460=1730$$

$$H^2-161H+4730=0$$

The solution of this quadratic is:

$$H=\frac{161\pm\sqrt{161^2-18920}}{2}$$

$$=\frac{161\pm 83{\cdot}7}{2}$$

$$=121{\cdot}9 \text{ or } 38{\cdot}7$$

Only the second solution is possible and therefore the mercury stands at a height of 38·7 cm. at 40° C.

91.

A closed vessel containing air, water vapour and water is heated from 5° to 40° C., the pressure within the vessel thereby rising from 72·15 cm. to 86·01 cm. Find the saturated vapour pressure of water at 40° C. if its value at 5° C. is 6·5 mm.

As the vessel is closed and contains water, we may assume that the total volume remains constant and that the enclosed air remains saturated with water vapour throughout.

At 5°, the partial pressure of water vapour

$$=0{\cdot}65 \text{ cm.}$$

and the partial pressure of air, therefore

$$=72{\cdot}15-0{\cdot}65$$

$$=71{\cdot}50 \text{ cm.}$$

The pressure of the air, heated from 5° to 40° at constant volume, becomes

$$71{\cdot}5\times\left(\frac{273+40}{273+5}\right)$$

$$=71{\cdot}5\times\frac{313}{278}$$

$=80{\cdot}50$ cm.

Since the total pressure at 40° $=86{\cdot}01$ cm.
the partial pressure of water vapour $=86{\cdot}01-80{\cdot}50$
$=5{\cdot}51$ cm.

Therefore the saturated vapour pressure of water at 40° C. $=5{\cdot}51$ cm.

92.

Calculate the mass of a litre of moist hydrogen collected over water at 15° C. when the height of the barometer is 76·5 cm. The density of hydrogen at N.T.P. is 0·000089 gm. per c.c., and that of water vapour is 9 times that of hydrogen, whilst the saturated pressure of water vapour at 15° C. is 1·27 cm.

As the gas has bubbled through water, it may be assumed to be saturated with water vapour at a partial pressure of 1·27 cm. The partial pressure of dry hydrogen is therefore

$$(76{\cdot}5-1{\cdot}27)=75{\cdot}23 \text{ cm.}$$

Mass of 1000 c.c. of hydrogen at 15° C. and 75·23 cm. pressure

$$=\left(0{\cdot}000089\times1000\times\frac{273}{288}\times\frac{75{\cdot}23}{76}\right)\text{ gm.}$$

Mass of 1000 c.c. of water vapour at 15° C. and 1·27 cm. pressure

$$=\left(9\times0{\cdot}000089\times1000\times\frac{273}{288}\times\frac{1{\cdot}27}{76}\right)\text{ gm.}$$

∴ Total mass of moist hydrogen

$$=\left(0{\cdot}089\times\frac{273}{288}\times\frac{75{\cdot}23}{76}\right)+\left(9\times0{\cdot}089\times\frac{273}{288}\times\frac{1{\cdot}27}{76}\right)\text{ gm.}$$

$$=\left(0{\cdot}089\times\frac{273}{288}\right)\left(\frac{75{\cdot}23}{76}+\frac{9\times1{\cdot}27}{76}\right)\text{ gm.}$$

$$=\left(0{\cdot}089\times\frac{273}{288}\times\frac{86{\cdot}66}{76}\right)\text{ gm.}$$

$$=0{\cdot}0962 \text{ gm.}$$

93.

Calculate the mass of a litre of air at 20° C. and 75·0 cm. pressure, the relative humidity being 60 per cent.

Saturated vapour pressure of water at 20° C.
$=1{\cdot}75$ cm.
Density of water-vapour at 0° C. and 76 cm. pressure
$=0{\cdot}806$ gm. per litre
Density of air at 0° C. and 76 cm. pressure $=1{\cdot}293$ gm. per litre

Since the relative humidity is 60 per cent., the pressure of water-vapour present in the air is $1{\cdot}75\times 0{\cdot}60=1{\cdot}050$ cm.

By the law of partial pressures, the mass of a litre of moist air at 75 cm. pressure is equal to:

$$\left(\begin{matrix}\text{Mass of a litre of water-vapour}\\ \text{at } 1{\cdot}05 \text{ cm. pressure}\end{matrix}\right)+\left(\begin{matrix}\text{Mass of a litre of dry air}\\ \text{at } (75-1{\cdot}05) \text{ cm. pressure}\end{matrix}\right)$$

$$=\left(0{\cdot}806\times\frac{1{\cdot}05}{76}\times\frac{273}{293}\right)+\left(1{\cdot}293\times\frac{73{\cdot}95}{76}\times\frac{273}{293}\right)\text{ gm.}$$

$$=\frac{273}{76\times 293}[(0{\cdot}806\times 1{\cdot}05)+(1{\cdot}293\times 73{\cdot}95)]$$

$$=\frac{273}{76\times 293}(0{\cdot}846+95{\cdot}60)$$

$$=\underline{1{\cdot}182\text{ gm.}}$$

94.

A quantity of air at 20° C. and 76 cm. pressure is suddenly compressed to half its original volume. Find the change in temperature, assuming no heat is gained or lost. The ratio of the specific heats of air $=1{\cdot}41$.

The adiabatic equation

(1) $$pv^{\gamma}=\text{const.}$$

and the general gas equation

(2) $$\frac{pv}{\mathrm{T}}=\text{const.}$$

may both be applied here.

Let the original volume be v and the final pressure p.

Then the final volume is $\frac{v}{2}$ and from (1):

$$76v^{1{\cdot}41}=p\left(\frac{v}{2}\right)^{1{\cdot}41}$$

$$\therefore \quad p = 76 \times 2^{1 \cdot 41}$$

$$\log p = 1{\cdot}8808 + 1{\cdot}41(0{\cdot}3010)$$
$$= 2{\cdot}3052$$
$$p = 201{\cdot}9 \text{ cm.}$$

$\therefore$ from (2):

$$\frac{76v}{293} = \frac{201{\cdot}9 \times \frac{v}{2}}{293 + t}$$

where t is the temperature rise.

$$\therefore \quad 293 + t = \frac{293 \times 201{\cdot}9}{2 \times 76}$$
$$= 389{\cdot}3$$
$$t = \underline{96{\cdot}3° \text{ C.}}$$

The temperature will rise by 96·3 C.

95.

Find the value of the gas constant R in the equation $pv = RT$ if v is the volume of 1 gm. molecule.

Density of hydrogen = 0·0899 gm./litre at N.T.P.
Density of mercury = 13·59 gm./c.c.

At N.T.P. $T = 273°$

$$p = (76 \times 13{\cdot}59 \times 981) \text{ dyne/cm.}^2$$

$$v = \text{volume of 2 gms. of } H_2 = \frac{2}{0{\cdot}0000899} \text{ c.c.}$$

$$R = \frac{pv}{T} = \left(\frac{76 \times 13{\cdot}59 \times 981 \times 2}{0{\cdot}0000899 \times 273}\right) \frac{\text{dyne}}{\text{cm.}^2} \cdot \frac{\text{cm.}^3}{\text{gm. mol.}} \cdot \frac{1}{°\text{C.}}$$
$$= \underline{8{\cdot}253 \times 10^7 \text{ ergs/gm. mol. deg. C.}}$$

96.

Deduce a value for J, the mechanical equivalent of heat, from the following data:

Specific heat of hydrogen at constant pressure = 3·402
Specific heat of hydrogen at constant volume = 2·402
Density of hydrogen at N.T.P. = 0·0899 gm. per litre.
Density of mercury = 13·59 gm. per c.c.

The specific heat of a gas at constant pressure exceeds that at constant volume by the thermal equivalent of the work done in

expanding 1 gm. of the gas through 1° C. against the constant pressure.

$$\text{That is: } C_p - C_v = \frac{p(v_1 - v_0)}{J}$$

where v_0 and v_1 are the specific volumes (volumes of 1 gm.) taken for convenience for expansion from 0° C. to 1° C.

At constant pressure

$$\frac{v_1}{v_0} = \frac{T_1}{T_0} = \frac{274}{273} \text{ or } v_1 = v_0 \times \frac{274}{273}$$

$$\therefore \qquad C_p - C_v = \frac{pv_0}{J}\left(\frac{274}{273} - 1\right)$$

$$= \frac{pv_0}{J}\left(\frac{274-273}{273}\right)$$

$$= \frac{pv_0}{J \times 273}$$

$$= \frac{p}{J \times 273 \times \rho_0}$$

since specific volume

$$= \frac{1}{\text{density}}$$

$$\therefore \qquad J = \frac{p}{273\rho_0(C_p - C_v)}$$

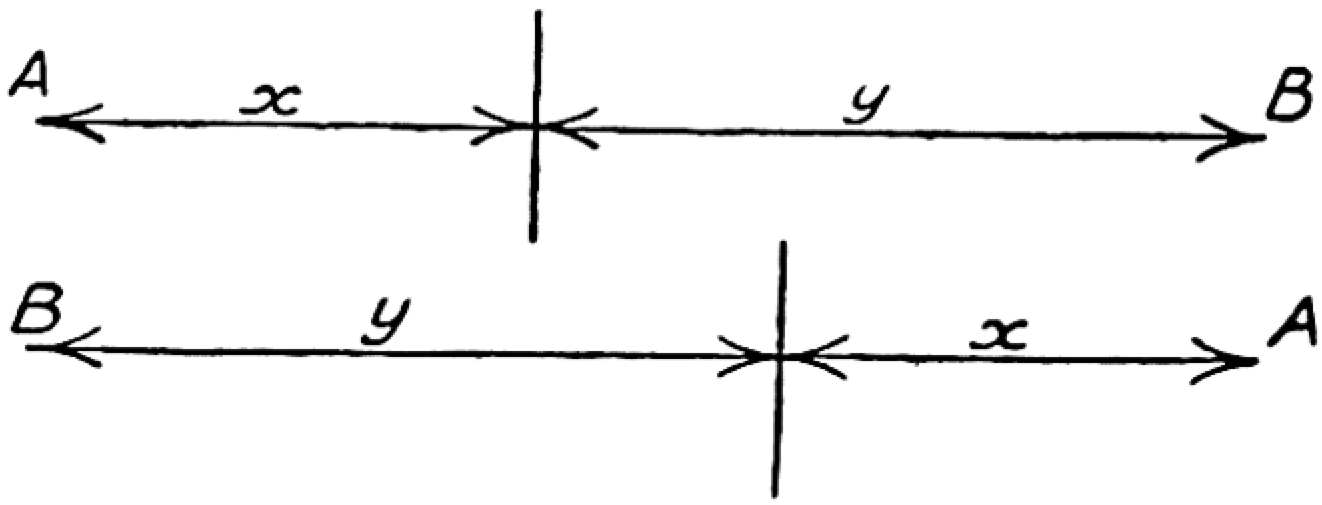
p is the pressure (in dynes per sq. cm.) at which ρ_0 is measured, the equivalent of 76 cm. of mercury, since ρ_0 is given at N.T.P.

$$\therefore \qquad J = \frac{76 \times 981 \times 13{\cdot}59}{273 \times 0{\cdot}0000899 \times 1{\cdot}000} \text{ ergs per cal.}$$

$$= \underline{4{\cdot}127 \times 10^7 \text{ ergs per cal.}}$$

97.

Two light sources are placed 150 cm. apart and a Bunsen grease-spot photometer screen is adjusted to such a position on the straight line joining them that the grease-spot disappears. The two sources are now interchanged and it is found that the screen has to be moved through 30 cm. to cause the spot to disappear again. What is the ratio of the intensities of the sources?



If x and y are the respective distances of the two sources from the screen in the first position, they must be y and x respectively in the second position. Therefore the distance moved through by the screen is $(y-x)$.

So $$y-x=30$$
and $$y+x=150$$
$\therefore$ $$(y+x)-(y-x)=150-30$$
$$2x=120$$
$$x=60 \text{ cm.}$$
$$y=90 \text{ cm.}$$

and the ratio of the intensities by the inverse square law is

$$\frac{y^2}{x^2}=\frac{90^2}{60^2}$$
$$=\frac{3^2}{2^2}$$
$$=\frac{9}{4}$$
$$=\underline{2{\cdot}25}$$

98.

In a Rumford photometer the shadows cast by a candle flame and an electric filament lamp are equally illuminated when their distances from the screen are respectively 20 and 100 cm. A sheet of glass is now held in front of the electric lamp, which has to be moved 8 cm. nearer to the screen in order to restore equality of illumination of the two shadows. How much nearer must the lamp be moved if another similar sheet of glass is added to the first?

If one thickness of glass transmits a fraction p of the light falling on it, two thicknesses will transmit a fraction p of p—that is, p^2.

By the inverse square law:

$$\text{Intensity of lamp} = \frac{100^2}{20^2} = 25 \text{ candle power}$$

Apparent intensity of lamp through one thickness of glass

$$=25p=\frac{92^2}{20^2}=21{\cdot}16 \text{ c.p.}$$

$$\therefore \quad p=0{\cdot}846$$

and $$p^2=0{\cdot}716$$

Apparent intensity of lamp through two thicknesses of glass

$$=25p^2$$

$$=25\times0{\cdot}716=\frac{d^2}{20^2}$$

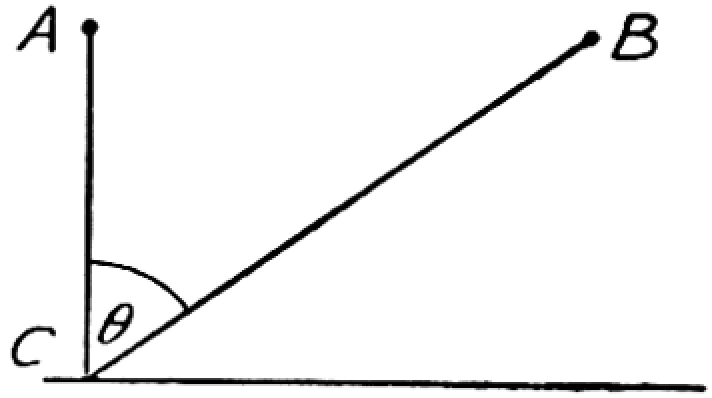
where d is the final distance of the lamp from the screen.

$$\therefore \quad d^2=400\times25\times0{\cdot}716$$
$$=7160$$
$$d=84{\cdot}6 \text{ cm.}$$

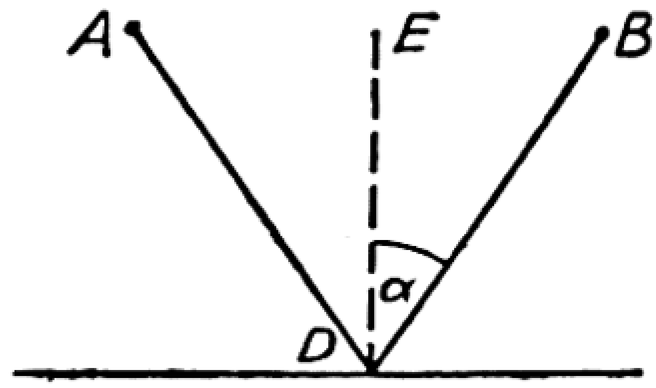
$\therefore$ The lamp must therefore be moved 7·4 cm. nearer to the screen.

99.

Two lamps, each of 300 candle power, are suspended 30 ft. above the ground and 40 ft. apart. Calculate the intensity of illumination on the ground (*a*) immediately beneath one of the lamps, (*b*) midway between the lamps.



(a)



(b)

(*a*) The illumination at C due to lamp A

$$=\frac{300}{AC^2}$$

$$=\frac{300}{30^2}\text{ ft. candles}$$

The illumination at C due to lamp B

$$=\frac{300}{CB^2}\cos\theta$$

$$=\frac{300}{CB^2}\cdot\frac{AC}{CB}$$

$$=\frac{300\times 30}{(\sqrt{30^2+40^2})^3}\text{ ft. candles}$$

$$=\frac{9000}{50^3}\text{ ft. candles}$$

$\therefore$ Total illumination at C due to both lamps

$$=\frac{300}{30^2}+\frac{9000}{50^3}$$

$$=\frac{1}{3}+\frac{9}{125}$$

$$=0{\cdot}333+0{\cdot}072$$

$$=\underline{0{\cdot}405\text{ ft. candles}}$$

(*b*) Illumination at D due to each lamp

$$=\frac{300}{DB^2}\cos\alpha$$

$$=\frac{300\times ED}{DB^3}$$

$$=\frac{300\times 30}{(\sqrt{30^2+20^2})^3}\text{ ft. candles}$$

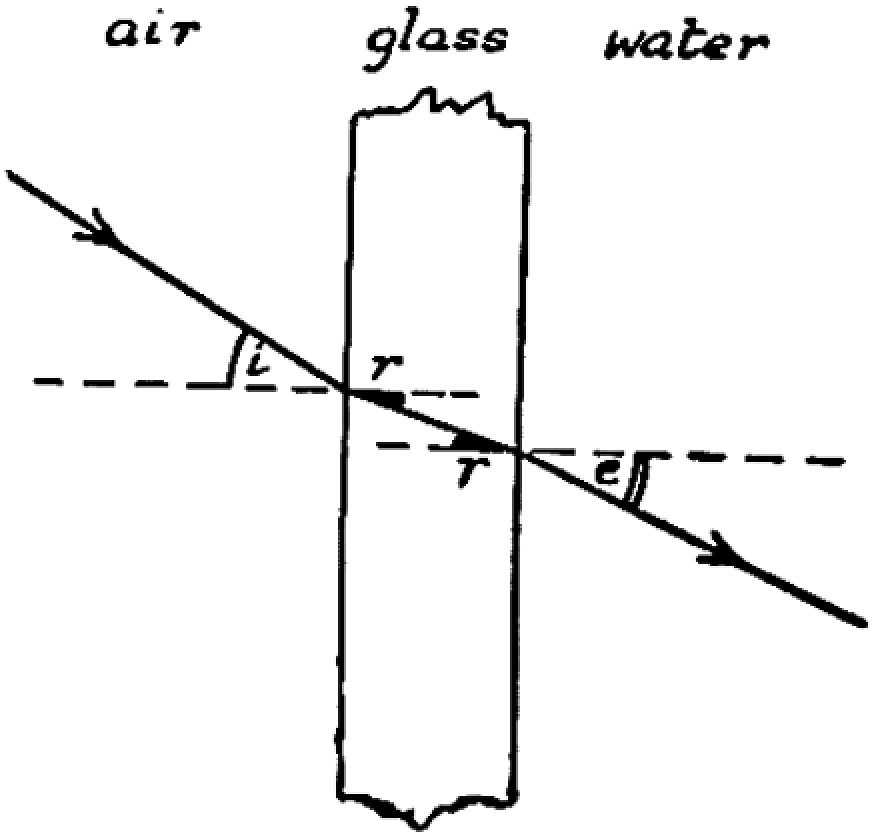
$$=\frac{9000}{46870}$$

$$=0{\cdot}192\text{ ft. candles}$$

$\therefore$ Illumination at D due to both lamps $=0{\cdot}384$ ft. candles.

100.

A ray of light passes through the side of a glass tank containing water. If the angle of incidence on the glass is 30°, what is the deviation of the ray at each refraction? (Refractive indices of glass and water, 1·50 and 1·33 respectively.)



At the first refraction, the ray passes from air to glass, and therefore, by Snell's Law,

$$1{\cdot}50=\frac{\sin i}{\sin r}$$

$$=\frac{\sin 30^\circ}{\sin r}$$

$$\sin r=\frac{\sin 30^\circ}{1{\cdot}50}=\frac{0{\cdot}50}{1{\cdot}50}$$

$$=0{\cdot}333$$

$$\therefore \quad r=19^\circ\ 28'$$

The deviation at the first refraction $=(i-r)$

$$=(30^\circ-19^\circ\ 28')$$

$$=10^\circ\ 32$$

The second refraction is from glass to water, for which we require ${}_g\mu_w$.

$${}_g\mu_w={}_g\mu_a\times{}_a\mu_w$$

$$=\frac{{}_a\mu_w}{{}_a\mu_g}$$

$$=\frac{1{\cdot}33}{1{\cdot}50}$$

$$=0{\cdot}887$$

The angle of incidence on the glass-water surface is equal to r, the angle of refraction at the air-glass surface. So, by Snell's Law,

$$0{\cdot}887=\frac{\sin r}{\sin e}=\frac{0{\cdot}333}{\sin e}$$

$$\sin e=\frac{0{\cdot}333}{0{\cdot}887}=0{\cdot}375$$

$$e=22^\circ\ 1'$$

And the deviation at the second refraction is

$$(e-r)=(22^\circ\ 1'-19^\circ\ 18')$$
$$=\underline{2^\circ\ 33'}$$

101.

A fly, looking down vertically into a pond from 12 in. above the surface, sees a fish apparently 18 in. below the surface. What is the actual depth of the fish? What is the apparent height of the fly as seen by the fish?

The ratio of real to apparent depth of a medium (1) viewed from another medium (2) is ${}_2\mu_1$ for rays nearly normal to the surface of separation.

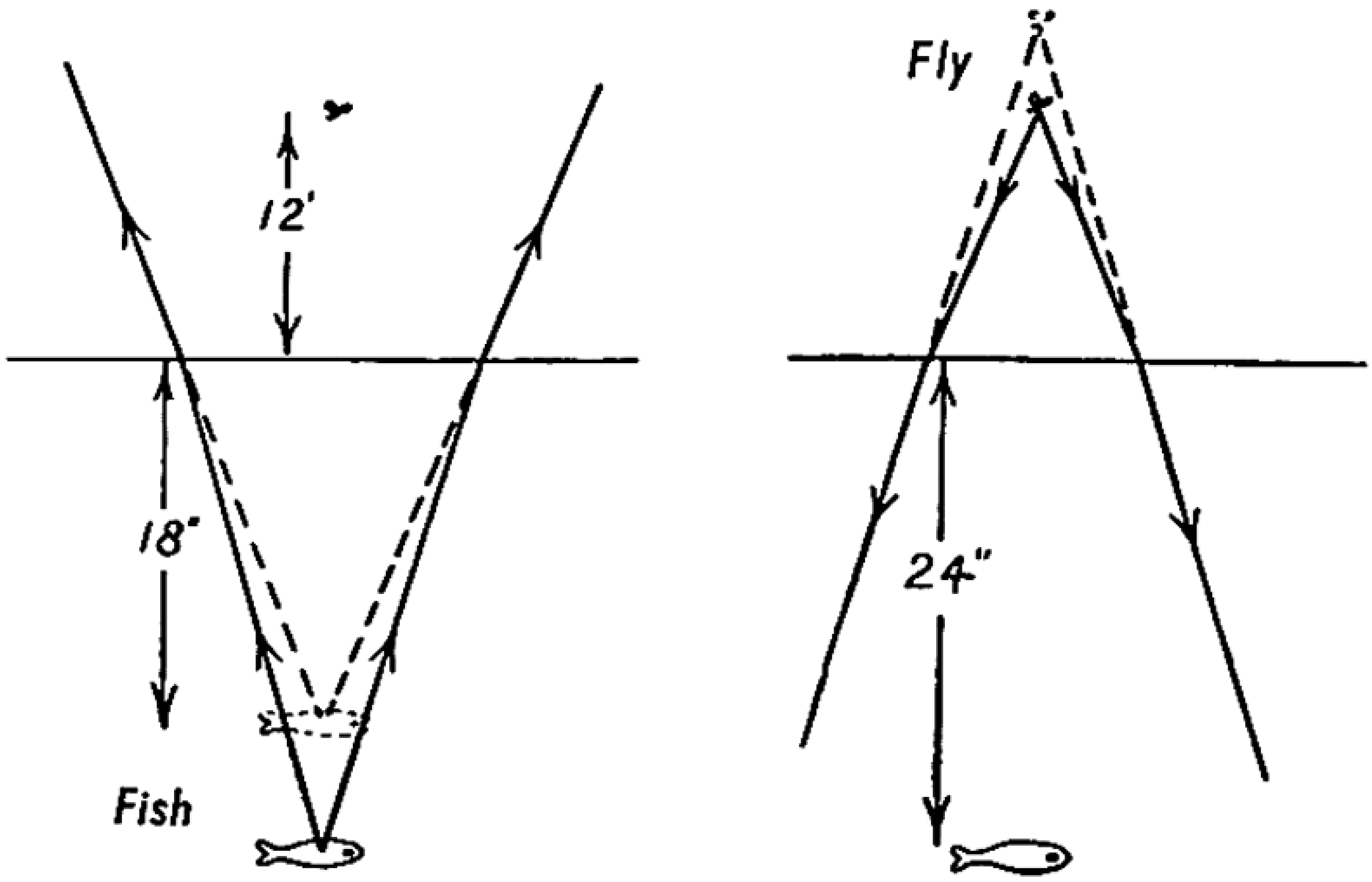
So, taking $\frac{4}{3}$ as the refractive index of water (${}_a\mu_w$), the real depth of the fish is

$$18\times\frac{4}{3}=\underline{24\text{ in.}}$$

Also since ${}_w\mu_a=\frac{1}{{}_a\mu_w}$, the apparent height of the fly above the water is

$$\frac{12}{\frac{3}{4}}=\underline{16\text{ in.}}$$

In the ray diagrams opposite, the angle of divergence of the rays has been exaggerated for the sake of clearness. Actually, only a narrow pencil of rays will be received by the eye in each case.



102.

Given that the refractive index for air-glass is 1·60 and for air-water is 1·33, calculate the critical angle for water-glass.

From the general expression for a number of different media:

$${}_1\mu_n = {}_1\mu_2 \times {}_2\mu_3 \times {}_3\mu_4 \ \cdots \ {}_{n-1}\mu_n$$

we may write $\quad {}_a\mu_g = {}_a\mu_w \times {}_w\mu_g$

$${}_w\mu_g = \frac{1{\cdot}60}{1{\cdot}33} = 1{\cdot}20$$

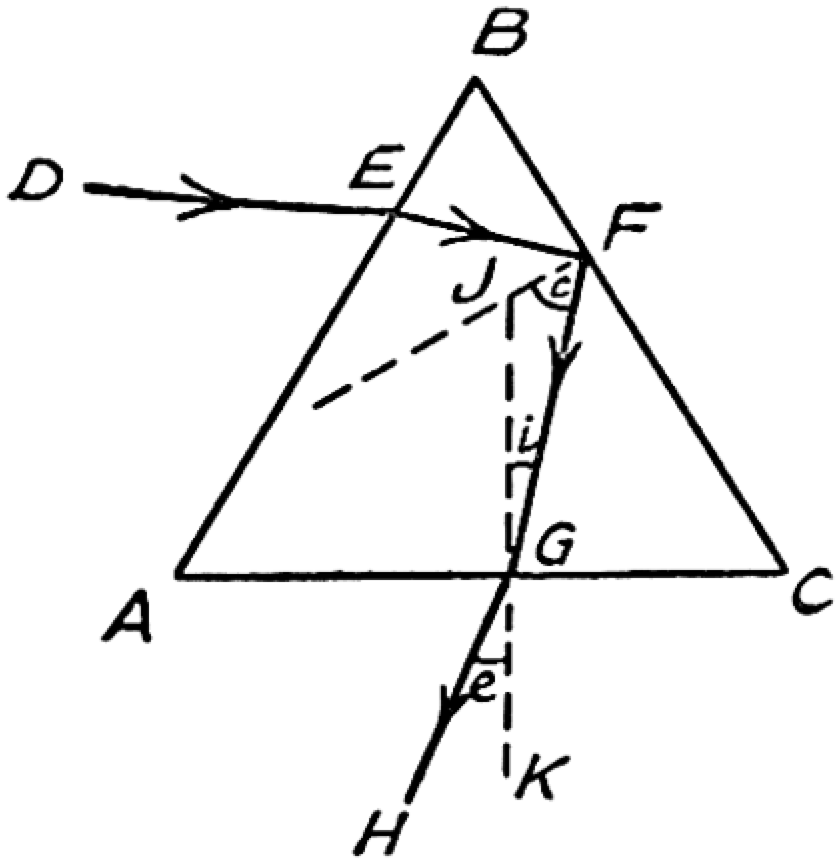
and since $\quad \sin {}_wc_g = \dfrac{1}{{}_w\mu_g}$ (c being the critical angle)

$$\sin {}_wc_g = \frac{1}{1{\cdot}2} = 0{\cdot}833$$

$\therefore \quad \underline{\underline{c = 56°\ 24}}$

103.

Light is incident on the face AB of a 60° glass prism ABC, is totally reflected at the face BC, and passes out through the face AC. What is the largest possible value of the angle of emergence? (Take μ for glass as 1·5.)



In the diagram, DEFGH represents the path of the ray through the prism, F being the point of total reflection. The normals through F and G intersect at J.

The maximum angle of emergence will be given by the minimum angle of total reflection at F, i.e. by the critical angle at F. Let us then call angle JFG, c.

We may obtain a value for the angle of emergence e (KGH) in terms of c by relating i (JGF) to each of them, thus:

In the triangle JFG, angle FJG$=120°$ (it is the supplement of $60°$)

$$\begin{aligned} i &= 180-(120°+c) \\ &= 60°-c \end{aligned}$$

For the refraction at G (glass to air)

$$\frac{\sin i}{\sin e} = {}_g\mu_a = \frac{1}{1{\cdot}5}$$

$$\therefore \quad \begin{aligned} \sin e &= 1{\cdot}5 \sin i \\ &= 1{\cdot}5 \times \sin(60°-c) \end{aligned}$$

c may be evaluated from the relation

$$\sin c = \frac{1}{{}_a\mu_g} = \frac{1}{1{\cdot}5} = 0{\cdot}667$$

giving $\quad c = 41°\ 48'$

$$\therefore \quad \begin{aligned} \sin e &= 1{\cdot}5 \times \sin(60°-41°\ 48') \\ &= 1{\cdot}5 \times \sin 18°\ 12' \\ &= 0{\cdot}468 \\ e &= 27°\ 54' \end{aligned}$$

<u>The maximum angle of emergence is 27° 54′.</u>

104.

What is the angle of minimum deviation for a ray passing through a 60° prism of refractive index 1·620?

The relation between the refractive index, the angle of the prism a, and the angle of minimum deviation θ is

$$\mu=\frac{\sin \frac{1}{2}(a+\theta)}{\sin \frac{1}{2}a}$$

Substituting the values given in this expression:

$$1{\cdot}620=\frac{\sin \frac{1}{2}(60°+\theta)}{\sin 30°}$$

$$\sin \tfrac{1}{2}(60°+\theta)=0{\cdot}5\times 1{\cdot}620$$

$$=0{\cdot}810$$

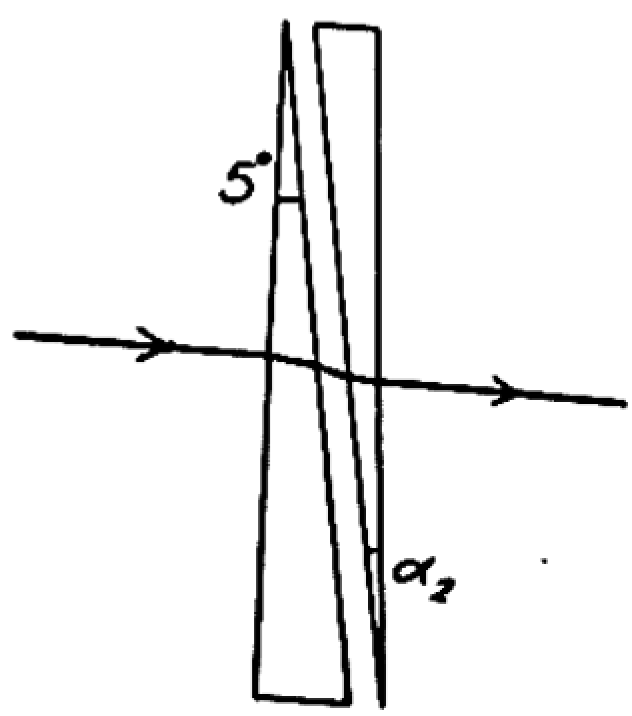
$$\tfrac{1}{2}(60°+\theta)=54°\ 6'$$

$$\theta=(2\times 54°\ 6')-60°$$

$$=\underline{48°\ 12'}$$

105.

A prism of angle 5°, made from glass of refractive index 1·52, is placed in contact with another prism made from glass of refractive index 1·63, and it is found that a ray of light passes normally through the two prisms without deviation. What is the angle of the second prism?



The minimum deviation relation,

$$\mu=\frac{\sin \frac{1}{2}(a+\theta)}{\sin \frac{1}{2}a}$$

becomes $\mu=\dfrac{a+\theta}{a}$

for a prism of small angle

whence $\mu=1+\dfrac{\theta}{a}$

and $\theta=a(\mu-1)$

θ no longer being the angle of minimum deviation, but the angle of deviation which remains constant over a wide range of values of the angle of incidence.

In this problem, the deviation produced by the first prism is neutralised by that produced by the second.

Therefore

$$\theta_1=\theta_2 \text{ and } \alpha_1(\mu_1-1)=\alpha_2(\mu_2-1)$$

$$5(1{\cdot}52-1)=\alpha_2(1{\cdot}63-1)$$

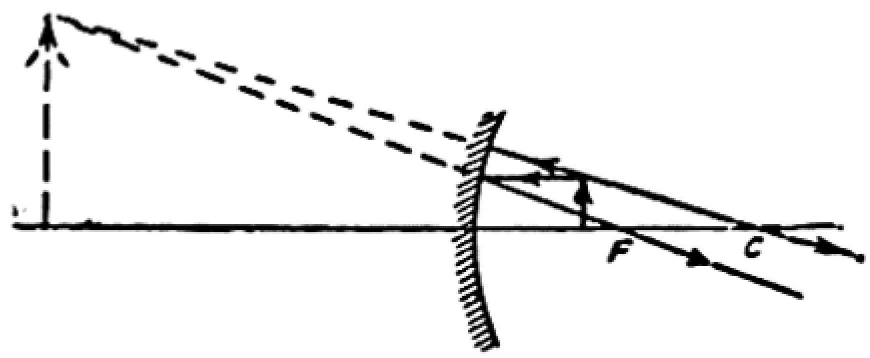
$$\alpha_2=\frac{5\times 0{\cdot}52}{0{\cdot}63}=4{\cdot}13°$$

$$=\underline{4°\ 8'}$$

106.

Determine the position and size of the image of an object 1 in. high, placed at distances of 6 and 12 in. respectively from a concave mirror of radius of curvature 16 in.

(*a*) $\frac{1}{v}+\frac{1}{u}=\frac{2}{R}$



$$\frac{1}{v}+\frac{1}{6}=\frac{2}{16}$$

$$\frac{1}{v}=\frac{1}{8}-\frac{1}{6}$$

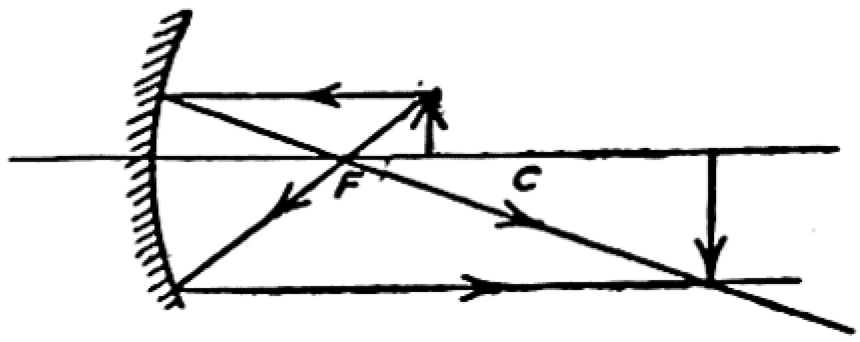
$$=-\frac{1}{24} \qquad \therefore \qquad \underline{v=-24 \text{ in.}}$$

$$\frac{I}{O}=\frac{v}{u}$$

$$\frac{I}{1}=\frac{-24}{6} \qquad \therefore \quad \underline{I=4 \text{ in.}} \text{ (ignoring the minus sign*)}$$

So the image is 24 in. behind the mirror (and is therefore virtual) and is 4 in. high.

(*b*) $\frac{1}{v}+\frac{1}{12}=\frac{2}{16}$



$$\frac{1}{v}=\frac{1}{8}-\frac{1}{12}$$

$$=\frac{1}{24}$$

$$\therefore \qquad \underline{v=24 \text{ in.}}$$

$$\frac{I}{1}=\frac{24}{12} \qquad \therefore \qquad \underline{I=2 \text{ in.}}$$

* The sign of $\frac{v}{u}$ may be taken as indicating whether the image is erect (negative) or inverted (positive). Confusion may arise, however, when lenses are considered, since the rule must then be reversed. On the whole, it is probably best to ignore the sign.

In this case the image is 24 in. in front of the mirror (and is therefore real) and is 2 in. high.

107.

At what distance from a concave mirror of focal length 10 cm. must an object be placed in order that an image double its size may be obtained?

There are two possibilities, the image being either (*a*) real or (*b*) virtual.

(*a*) Since $\frac{I}{O}=\frac{v}{u}$ and v must be positive for a real image formed by a mirror.

$$\therefore \qquad 2=\frac{v}{u} \text{ and } v=2u$$

Hence, substituting in the mirror equation, we have

$$\frac{1}{2u}+\frac{1}{u}=\frac{1}{10}$$

$$\frac{1+2}{2u}=\frac{1}{10}$$

$$\underline{u=15 \text{ cm.}}$$

(*b*) For a virtual image v must have a negative sign and therefore

$$2=\frac{-v}{u}$$

$$v=-2u$$

$$\therefore \qquad -\frac{1}{2u}+\frac{1}{u}=\frac{1}{10}$$

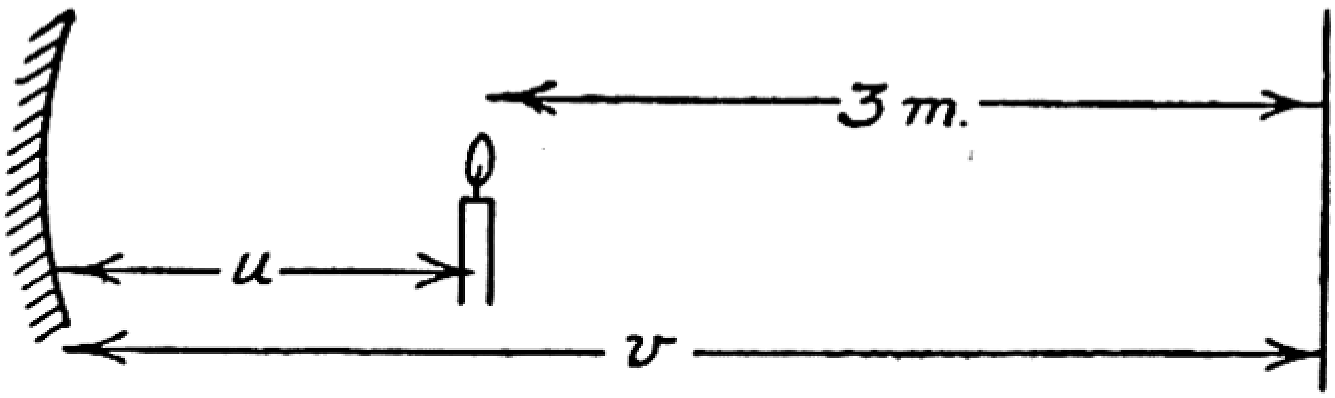
$$\frac{1}{2u}=\frac{1}{10}$$

$$\underline{u=5 \text{ cm.}}$$

The object must therefore be placed 15 cm. from the mirror to obtain a real image of double its size, and 5 cm. from the mirror for a virtual image of double its size.

108.

A candle flame 3 cm. high is placed at a distance of 3 metres from a wall. How far from the wall must a concave mirror be placed in order that it may form an image of the flame 9 cm. high on the wall? What must be the radius of curvature of the mirror?



From the diagram it is seen that

$$v-u=300 \text{ cm.}$$

Also, from the expression $\dfrac{I}{O}=\dfrac{v}{u}$

we have $$\frac{9}{3}=\frac{v}{u}$$

or $$v=3u$$

Substituting this value of v in the first equation gives

$$3u-u=300 \text{ cm.}$$

$\therefore$ $$u=150 \text{ cm.}$$

and $$\underline{v=450 \text{ cm.}}$$

$$\frac{1}{v}+\frac{1}{u}=\frac{2}{R}$$

$\therefore$ $$\frac{1}{450}+\frac{1}{150}=\frac{2}{R}$$

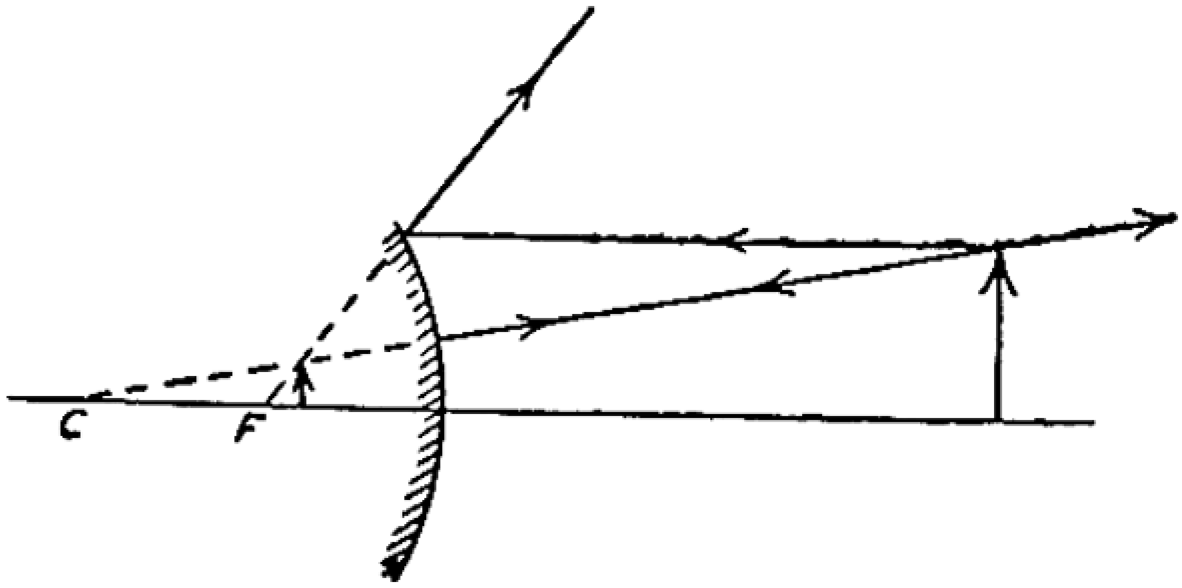
$$\frac{4}{450}=\frac{2}{R}$$

$$\underline{R=225 \text{ cm.}}$$

The mirror must be placed 450 cm. from the wall and must have a radius of curvature of 225 cm.

109.

An object 4 cm. high is placed at a distance of 15 cm. in front of a convex mirror having a radius of curvature of 10 cm. Where will the image be and what will be its height?



$$\frac{1}{v}+\frac{1}{u}=\frac{2}{R}$$

$$\frac{1}{v}+\frac{1}{15}=\frac{2}{-10}$$

$$\frac{1}{v}=-\frac{1}{5}-\frac{1}{15}$$

$$=-\frac{4}{15}$$

$$\therefore\quad \underline{v=-3{\cdot}75 \text{ cm.}}$$

$$\frac{I}{O}=\frac{v}{u}$$

$$\frac{I}{4}=-\frac{3{\cdot}75}{15}$$

$\therefore$ $\underline{I=1 \text{ cm.}}$ (ignoring the minus sign*)

The image is therefore 3·75 cm. behind the mirror (and is therefore virtual, as always with a convex mirror) and is 1 cm. high.

110.

Two mirrors, one concave, the other convex, each of radius of curvature 20 cm., are placed coaxially 40 cm. apart, with their polished surfaces facing each other, and an object is placed midway between them. Find the position of the image formed by reflection first at the convex, then at the concave surface.

For the *first reflection* (at the convex mirror)

$$u_1=20 \text{ cm.}$$
$$R_1=-20 \text{ cm.}$$

$$\frac{1}{v_1}+\frac{1}{20}=-\frac{2}{20}$$

$$\frac{1}{v_1}=-\frac{3}{20}$$

$$\underline{v=-6\tfrac{2}{3} \text{ cm.}}$$

* See footnote to Ex. 97.

The first image is therefore a virtual one, $6\frac{2}{3}$ cm. behind the convex mirror.

For the *second reflection* (at the concave mirror) the first image constitutes the object and therefore

$$u_2=(40+6\tfrac{2}{3})\text{ cm.}$$

$$R_2=20\text{ cm.}$$

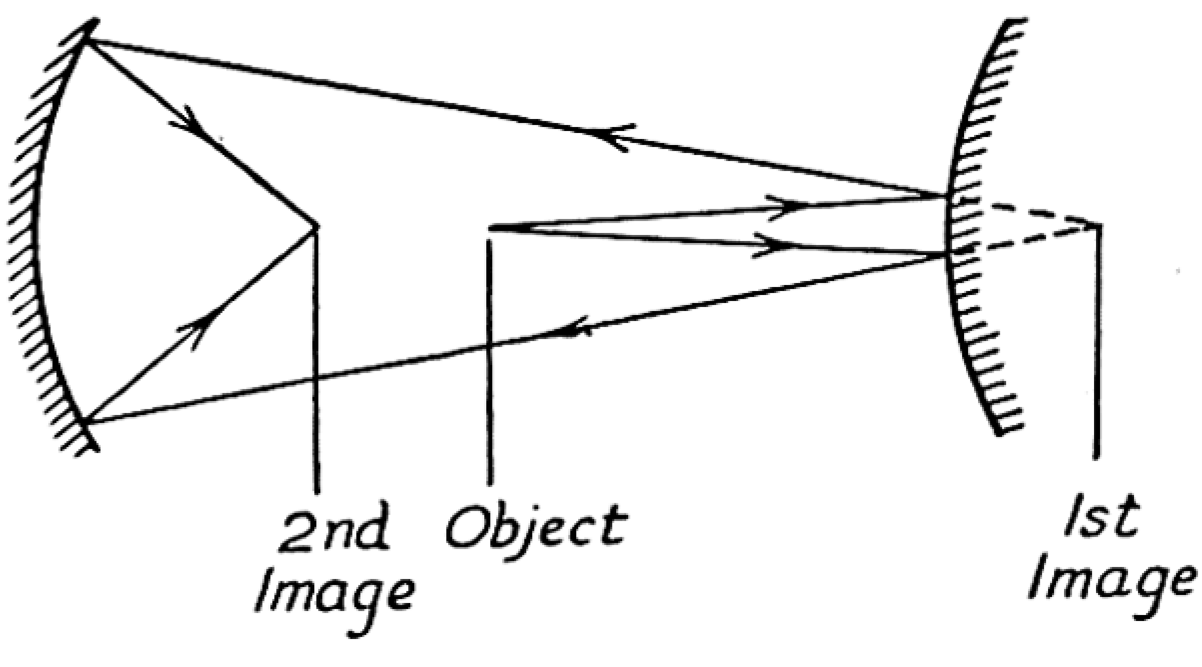
$$\frac{1}{v_2}+\frac{1}{46\frac{2}{3}}=\frac{2}{20}$$

$$\frac{1}{v_2}=\frac{2}{20}-\frac{3}{140}$$

$$=\frac{14-3}{140}$$

$$\underline{v_2=12\tfrac{8}{11}\text{ cm.}}$$

So the second image is real and is formed at a distance of $12\frac{8}{11}$ cm. in front of the concave mirror.



111.

A slab of glass of thickness 5 cm. and refractive index 1·5 is held a few centimetres in front of a concave mirror of radius of curvature 40 cm., the faces of the slab being perpendicular to the principal axis of the mirror. How far from the mirror must a small object be placed if its reflected image coincides with the object?

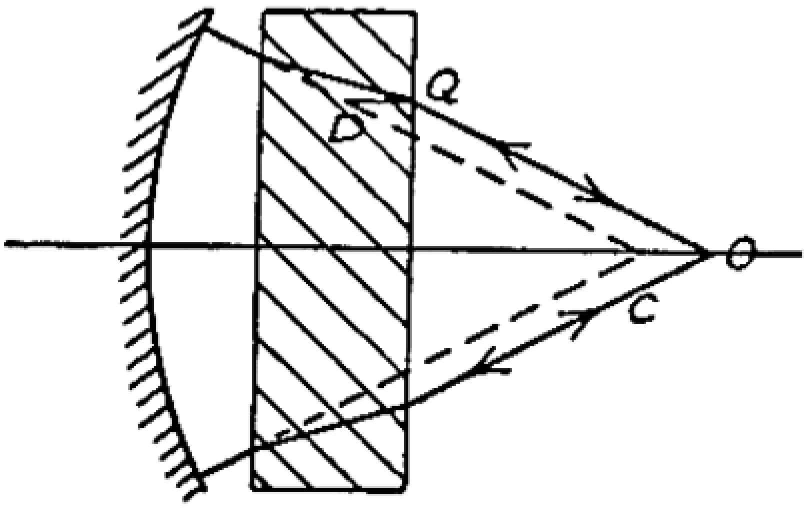


Fig. 1.

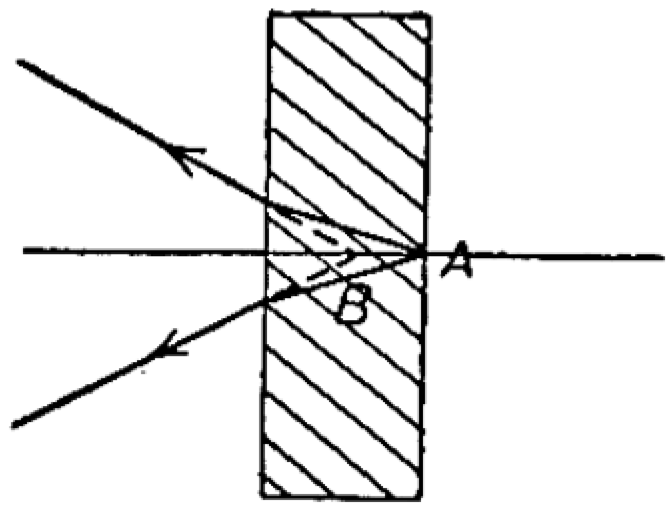


Fig. 2.

Fig. 1 shows the paths of rays diverging from the object O and converging, after reflection and refraction, to the same point. Fig. 2 shows the familiar "real and apparent thickness" diagram (see Ex. 92), B being the image of A. The distance QD is equal to AB which is the difference between real and apparent thicknesses.

$$\therefore \qquad QD=5-\frac{5}{\mu}$$

$$=5\left(1-\frac{1}{1\cdot5}\right)$$

$$=\frac{5\times0\cdot5}{1\cdot5}$$

$$=1\tfrac{2}{3}\text{ cm.}$$

and since OCDQ is a parallelogram,

$$OC=QD$$

$$\therefore \qquad OC=1\tfrac{2}{3}\text{ cm.}$$

but C is the centre of curvature since the rays at X and Y retrace their incident paths after reflection and must therefore be normal to the mirror, i.e. must travel along radii of curvature.

Therefore the distance of O from the mirror is 40+OC

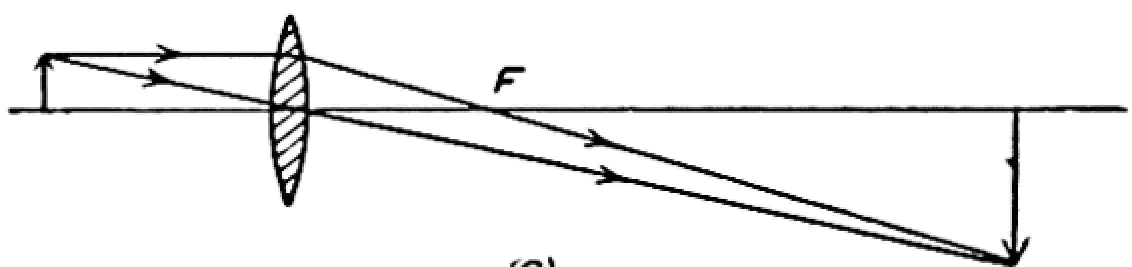
$$=\underline{41\tfrac{2}{3}\text{ cm.}}$$

112.

Find the position of the image formed by a convex lens of 20 cm. focal length, when an object is placed (*a*) 30 cm. from it, (*b*) 15 cm. from it.

I*	II*
$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$	$\frac{1}{f}=\frac{1}{v}+\frac{1}{u}$
For a convex lens, f is negative.	For a convex lens, f is positive.
(*a*) $u=30$ cm.	(*a*) $u=30$ cm.
$\frac{1}{-20}=\frac{1}{v}-\frac{1}{30}$	$\frac{1}{20}=\frac{1}{v}+\frac{1}{30}$
$\frac{1}{v}=\frac{1}{30}-\frac{1}{20}$	$\frac{1}{v}=\frac{1}{20}-\frac{1}{30}$
$=-\frac{1}{60}$	$=\frac{1}{60}$
$v=-60$ cm.	$v=60$ cm.

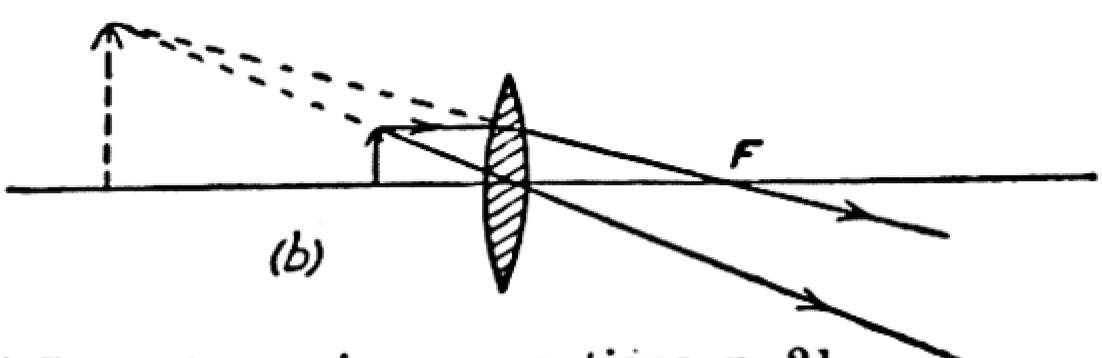
The image is real and is formed 60 cm. from the lens on the side opposite to the object.



(a)

(*b*) $u=15$ cm.	(*b*) $u=15$ cm.
$\frac{1}{-20}=\frac{1}{v}-\frac{1}{15}$	$\frac{1}{20}=\frac{1}{v}+\frac{1}{15}$
$\frac{1}{v}=\frac{1}{15}-\frac{1}{20}$	$\frac{1}{v}=\frac{1}{20}-\frac{1}{15}$
$=\frac{1}{60}$	$=-\frac{1}{60}$
$v=60$ cm.	$v=-60$ cm.

The image is virtual and is formed 60 cm. from the lens on the same side as the object.



(b)

* See note on sign conventions, p. 21.

113.

An object is placed on the principal axis of a concave lens of 12 in. focal length and an image is observed at a distance of 8 in. from the lens. How far is the object from the lens?

I

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$$

$f=12$ in.

$v=8$ in.

(since the image must be virtual with a concave lens)

$$\therefore \quad \frac{1}{12}=\frac{1}{8}-\frac{1}{u}$$

$$\frac{1}{u}=\frac{1}{8}-\frac{1}{12}$$

$$=\frac{3-2}{24}$$

$$\therefore \quad \underline{u=24 \text{ in.}}$$

II

$$\frac{1}{f}=\frac{1}{v}+\frac{1}{u}$$

$f=-12$ in.

$v=-8$ in.

(since the image must be virtual with a concave lens)

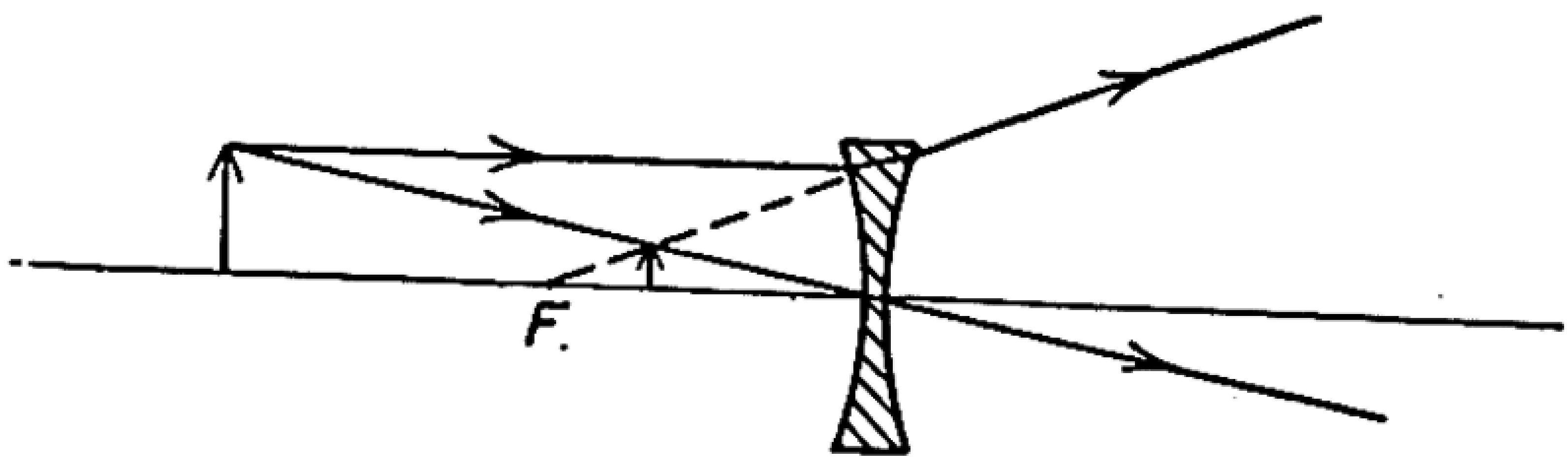
$$\therefore \quad -\frac{1}{12}=-\frac{1}{8}+\frac{1}{u}$$

$$\frac{1}{u}=\frac{1}{8}-\frac{1}{12}$$

$$=\frac{3-2}{24}$$

$$\therefore \quad \underline{u=24 \text{ in.}}$$

<u>The object must be placed 24 in. from the lens.</u>



114.

It is desired to project the image of a lantern slide 3 in. square on to a screen 5 ft. square at a distance of 20 ft. Of what focal length must the objective be?

Image magnification. $\frac{I}{O}=\frac{5\times 12}{3}=20$

I

As the image is real and v therefore negative:

$$\frac{v}{u}=-20$$

or $u=-\dfrac{v}{20}$

but $v=-(20\times 12)$ in.

$$\therefore\quad u=\frac{20\times 12}{20}$$
$$=12 \text{ in.}$$
$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$$
$$=-\frac{1}{240}-\frac{1}{12}$$
$$=-\frac{21}{240}$$
$$f=-\frac{240}{21}$$
$$=\underline{-11{\cdot}43 \text{ in.}}$$

II

$$\therefore\quad \frac{v}{u}=20$$

or $u=\dfrac{v}{20}$

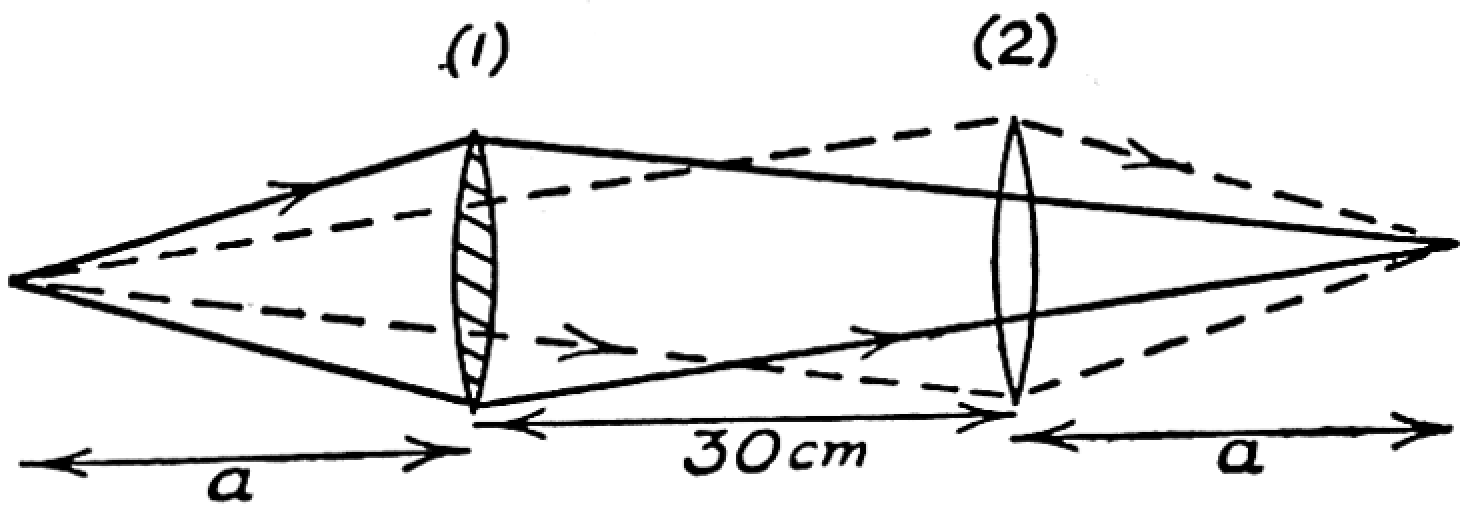
but $v=(20\times 12)$ in.

$$u=\frac{20\times 12}{20}$$
$$=12 \text{ in.}$$
$$\frac{1}{f}=\frac{1}{v}+\frac{1}{u}$$
$$=\frac{1}{240}+\frac{1}{12}$$
$$=\frac{21}{240}$$
$$=\frac{240}{21}$$
$$=\underline{11{\cdot}43 \text{ in.}}$$

<u>The focal length of the projection lens must be 11·43 in.</u>

115.

A convex lens is used to form an image of a lamp filament on a wall. It is then found that by moving the lens 30 cm. nearer to the wall a sharp image is again obtained, exactly one-quarter the length of the first. What is the focal length of the lens?



On referring to the diagram, it is seen that the object and image distances in position (1) are interchanged in position (2), the two distances being a and $(30+a)$ respectively. Substitution in the image magnification equation gives

$$\left.\begin{aligned} &(1) \qquad \frac{I_1}{O}=\frac{30+a}{a} \\ &(2) \qquad \frac{I_2}{O}=\frac{a}{30+a} \end{aligned}\right\} \frac{I_1}{I_2}=\frac{(30+a)^2}{a^2}=4$$

$$\frac{30+a}{a}=2$$

$$30+a=2a$$

$$a=30 \text{ cm.}$$

I

$\therefore \quad u=30$ cm.

$v=-60$ cm.

$$\frac{1}{f}=-\frac{1}{60}-\frac{1}{30}$$

$$=-\frac{3}{60}$$

$$\underline{f=-20 \text{ cm.}}$$

II

$\therefore \quad u=30$ cm.

$v=60$ cm.

$$\frac{1}{f}=\frac{1}{60}+\frac{1}{30}$$

$$=\frac{3}{60}$$

$$\underline{f=20 \text{ cm.}}$$

116.

How far must an object be placed from a lens of 15 cm. focal length in order that the area of the image formed by the lens may be 9 times the area of the object?

It is not stated whether the image is real or virtual or whether the lens is convex or concave. But as the image is larger than the object the possibility of the lens being concave is ruled out. We have therefore to consider two cases—*real* or *virtual* image. In both cases, since the area of the image is 9 times that of the object, the linear dimensions of the image are $\sqrt{9}$ times those of the object. That is:

$$\frac{I}{O}=3$$

(*a*) *Real image*

I

v is negative.

$$\therefore \quad \frac{v}{u} = -3 \text{ or } v = -3u$$

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$f = -15 \text{ cm.}$$

the lens being convex

$$\therefore \quad -\frac{1}{15} = \frac{1}{-3u} - \frac{1}{u}$$

$$\frac{1}{15} = \frac{1+3}{3u}$$

$$3u = 4 \times 15$$

$$\underline{u = 20 \text{ cm.}}$$

II

v is positive.

$$\therefore \quad \frac{v}{u} = e \text{ or } v = 3u$$

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$f = 15 \text{ cm.}$$

the lens being convex

$$\therefore \quad \frac{1}{15} = \frac{1}{3u} + \frac{1}{u}$$

$$\frac{1}{15} = \frac{1+3}{3u}$$

$$3u = 4 \times 15$$

$$\underline{u = 20 \text{ cm.}}$$

The object must be 20 cm. from the lens to produce a real image magnified 9 times superficially.

(*b*) *Virtual image*

v is positive.

$$v = 3u$$

$$-\frac{1}{15} = \frac{1}{3u} - \frac{1}{u}$$

$$= \frac{1-3}{3u}$$

$$3u = 15 \times 2$$

$$\underline{u = 10 \text{ cm.}}$$

v is negative.

$$v = -3u$$

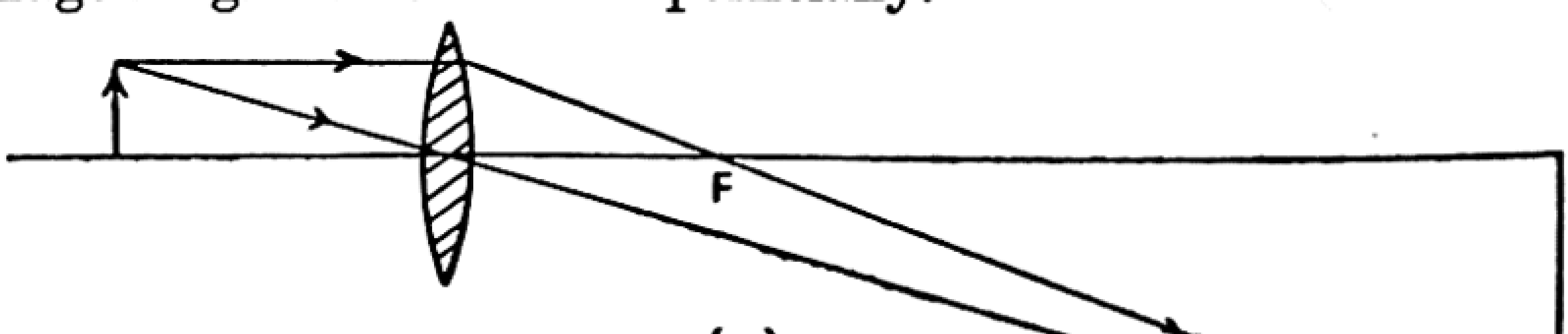
$$\frac{1}{15} = -\frac{1}{3u} + \frac{1}{u}$$

$$= \frac{-1+3}{3u}$$

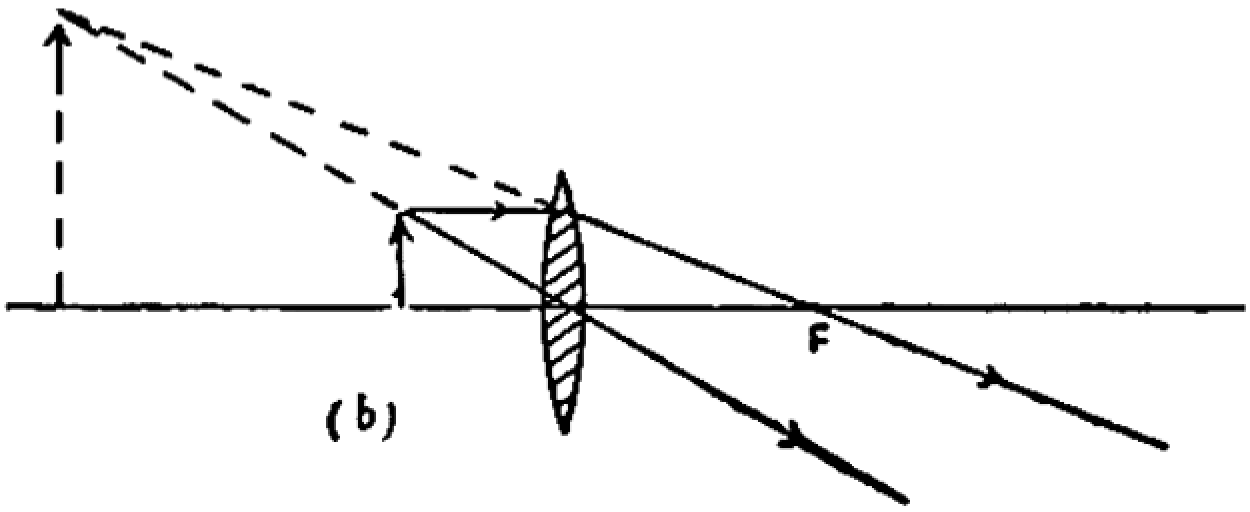
$$3u = 15 \times 2$$

$$\underline{u = 10 \text{ cm.}}$$

The object must be 10 cm. from the lens to produce a virtual image magnified 9 times superficially.



(a)



(b)

117.

A convex lens of focal length 25 cm. is placed in contact with a concave lens, and the combination produces a real image at a distance of 80 cm. when the object is at a distance of 40 cm. What is the focal length of the concave lens?

I

Calling the focal length of the combination F, we have

$$\frac{1}{F}=-\frac{1}{80}-\frac{1}{40}$$
$$=-\frac{3}{80}$$
$$F=-26\tfrac{2}{3}\text{ cm.}$$

The focal length of a combination of two lenses in contact is given by:

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}$$
$$f_1=-25\text{ cm.}$$
$$\therefore\quad -\frac{3}{80}=-\frac{1}{25}+\frac{1}{f2}$$
$$\frac{1}{f_2}=\frac{1}{25}-\frac{3}{80}$$
$$=\frac{16-15}{400}$$
$$=\frac{1}{400}$$
$$f_2=400\text{ cm.}$$

II

Calling the focal length of the combination F, we have

$$\frac{1}{F}=\frac{1}{80}+\frac{1}{40}$$
$$=\frac{3}{80}$$
$$F=26\tfrac{2}{3}\text{ cm.}$$

The focal length of a combination of two lenses in contact is given by:

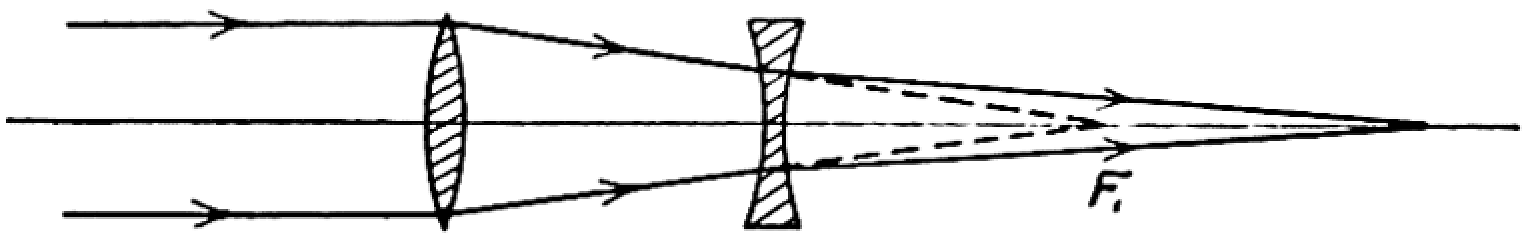
$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}$$
$$f_1=25\text{ cm.}$$
$$\therefore\quad \frac{3}{80}=\frac{1}{25}+\frac{1}{f_2}$$
$$\frac{1}{f_2}=\frac{3}{80}-\frac{1}{25}$$
$$=\frac{15-16}{400}$$
$$=-\frac{1}{400}$$
$$f_2=-400\text{ cm.}$$

The concave lens has a focal length of 400 cm.

118.

A compound lens consists of two thin lenses, one convex, the other concave, each of focal length 20 cm., placed 10 cm. apart on a common principal axis. Find the position of the principal focus of the combination when the light is incident first on (*a*) the convex lens, (*b*) the concave lens.

(*a*)



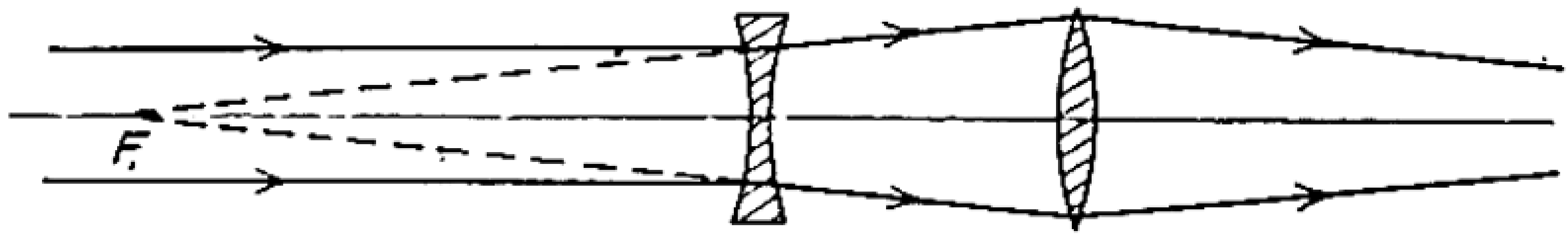
Parallel rays falling on to the convex lens are made to converge on to its principal focus at F_1 in the diagram, 10 cm. beyond the concave lens. The point F_1 may therefore be considered as a "virtual" object for the concave lens, the object distance u having a negative value of -10 cm.

So for the concave lens:

I	II
$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$	$\frac{1}{f}=\frac{1}{v}+\frac{1}{u}$
$\frac{1}{20}=\frac{1}{v}-\frac{1}{-10}$	$-\frac{1}{20}=\frac{1}{v}+\frac{1}{-10}$
$\frac{1}{v}=\frac{1}{20}-\frac{1}{10}$	$\frac{1}{v}=\frac{1}{10}-\frac{1}{20}$
$=-\frac{1}{20}$	$=\frac{1}{20}$
$v=-20$ cm.	$v=20$ cm.
The negative sign indicates a real image, which is the required principal focus, since it is the focus for parallel rays incident upon the combination.	The positive sign indicates a real image, which is the required principal focus, since it is the focus for parallel rays incident upon the combination.

<u>The principal focus of the compound lens is 20 cm. from the concave lens on the side remote from the object.</u>

(*b*)



Parallel rays falling on the concave lens will be made to diverge as from its principal focus F_1. The divergent beam incident upon the convex lens may therefore be considered as coming from F_1, 30 cm. away. So $u=30$ cm.

For the convex lens:

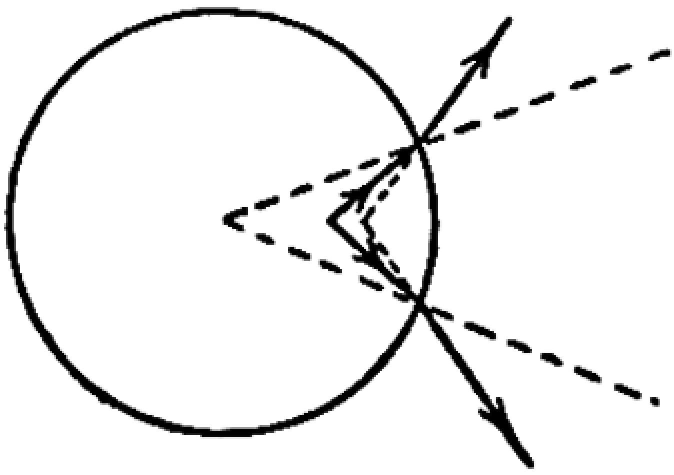
$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$	$\frac{1}{f}=\frac{1}{v}+\frac{1}{u}$
$-\frac{1}{20}=\frac{1}{v}-\frac{1}{30}$	$\frac{1}{20}=\frac{1}{v}+\frac{1}{30}$
$\frac{1}{v}=\frac{1}{30}-\frac{1}{20}$	$\frac{1}{v}=\frac{1}{20}-\frac{1}{30}$
$=\frac{-1}{60}$	$=\frac{1}{60}$
$v=-60$ cm.	$v=60$ cm.

In this case the principal focus of the combination is 60 cm. from the convex lens on the side remote from the object.

Note that this type of lens combination used as in (*a*) is known to photographers as a "telephoto lens."

119.

A sphere of glass, of refractive index 1·5 and radius 2 cm., contains a small air-bubble at a distance of 1 cm. from the centre. Find the position of the image of the bubble seen by an eye looking along a diameter (*a*) through the least and (*b*) through the greatest thickness of glass.



(*a*) The bubble is between the centre of the sphere and the eye.

I

Applying the equation

$$\frac{\mu}{v}-\frac{1}{u}=\frac{\mu-1}{R}$$

in which $\begin{cases} \mu={}_g\mu_a=\dfrac{1}{1\cdot5} \\ u=1 \text{ cm.} \\ R=2 \text{ cm.} \end{cases}$

we have

$$\frac{1}{1\cdot5v}-\frac{1}{1}=\frac{\frac{1}{1\cdot5}-1}{2}$$

$$\frac{1}{1\cdot5v}=\left(\frac{1-1\cdot5}{1\cdot5\times2}\right)+1$$

$$\frac{1}{v}=\frac{-0\cdot5}{2}+1\cdot5$$

$$=1\cdot25$$

$$\underline{v=0\cdot8 \text{ cm.}}$$

The image (virtual, because v has a positive sign) is therefore situated 0·8 cm. from the surface, the apparent position of the bubble being 0·2 cm. nearer to the surface than the true position.

II

Applying the equation

$$\frac{\mu}{v}+\frac{1}{u}=\frac{\mu-1}{R}$$

in which $\begin{cases} \mu={}_g\mu_a=\dfrac{1}{1\cdot5} \\ u=1 \text{ cm.} \\ R=-2 \text{ cm.} \end{cases}$

(Surface concave to the light) we have

$$\frac{1}{1\cdot5v}+\frac{1}{1}=\frac{\frac{1}{1.5}-1}{-2}$$

$$\frac{1}{1\cdot5v}=\left(\frac{1-1\cdot5}{-1\cdot5\times2}\right)-1$$

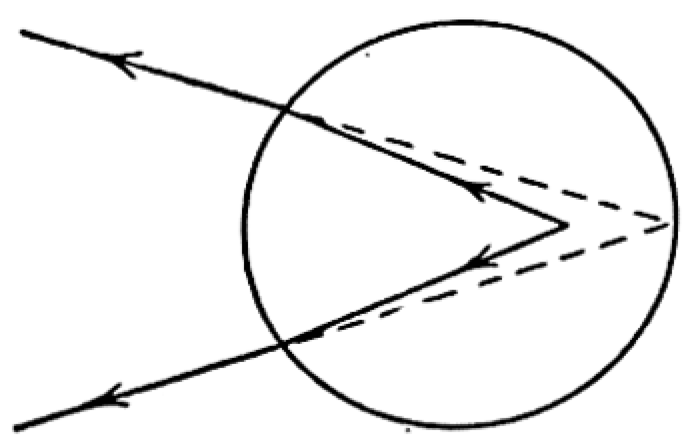
$$\frac{1}{v}=\frac{0\cdot5}{2}-1\cdot5$$

$$=-1\cdot25$$

$$\underline{v=-0\cdot8 \text{ cm.}}$$

The image (virtual, because v has a negative sign) is therefore situated 0·8 cm. from the surface, the apparent position of the bubble being 0·2 cm. nearer to the surface than the true position.

(*b*) The centre of the sphere is now between the bubble and the eye.



In this case

$$\begin{cases} \mu=\dfrac{1}{1\cdot5} \\ u=3 \text{ cm.} \\ R=2 \text{ cm.} \end{cases}$$

$$\therefore \quad \frac{1}{1\cdot5v}-\frac{1}{3}=\frac{\frac{1}{1\cdot5}-1}{2}$$

$$\frac{1}{v}=\frac{0\cdot5}{2}+\frac{1\cdot5}{3}$$

$$=0\cdot25$$

$$\underline{v=4 \text{ cm.}}$$

In this case

$$\begin{cases} \mu=\dfrac{1}{1\cdot5} \\ u=3 \text{ cm.} \\ R=-2 \text{ cm.} \end{cases}$$

$$\therefore \quad \frac{1}{1\cdot5v}+\frac{1}{3}=\frac{\frac{1}{1\cdot5}-1}{-2}$$

$$\frac{1}{v}=\frac{0\cdot5}{2}-\frac{1\cdot5}{3}$$

$$=-0\cdot25$$

$$\underline{v=-4 \text{ cm.}}$$

The image (again virtual) is now 4 cm. from the surface and is therefore in the further surface of the sphere.

If the bubble were at the centre of the sphere, its image would coincide with it; such would also be the case if the bubble were in the nearer surface. But when the bubble is between the centre of the sphere and the observer, as in (*a*), its image is nearer to the observer; whereas when the bubble is beyond the centre, as in (*b*), its image is further away.

120.

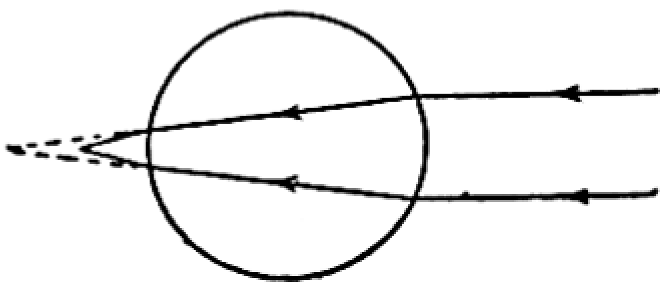
Find the position of the principal focus of a sphere of glass of refractive index 1·5 and radius 5 cm. for rays which pass close to the centre and make small angles with each other.

The restriction of the rays is simply a statement of the condition which must always apply when the simple lens formulæ are employed. With a lens having a unique principal axis, this condition is that the rays shall make small angles with the axis. The condition has to be worded differently in the case of a refracting sphere which has an infinitude of "principal axes"—straight lines passing through the centre. Failure to observe this condition necessitates the consideration of spherical aberration.

Consider a narrow beam of parallel rays incident on the sphere symmetrically with respect to a diameter. The focus of these rays after refraction at both surfaces of the sphere is the principal focus. Notice that at the first surface the light is passing from air to glass, and the value of μ to be used is ${}_a\mu_g$, whereas at the second surface it is ${}_g\mu_a$.

For the first refraction

I	II
$\frac{\mu}{v_1}-\frac{1}{u_1}=\frac{\mu-1}{R_1}$	$\frac{\mu}{v_1}+\frac{1}{u_1}=\frac{\mu-1}{R_1}$
$\frac{1\cdot5}{v_1}-\frac{1}{\propto}=\frac{1\cdot5-1}{-5}$	$\frac{1\cdot5}{v_1}+\frac{1}{\propto}=\frac{1\cdot5-1}{5}$
$v_1=-15$ cm.	$v_1=15$ cm.



This indicates a real image which would be formed at a distance of 5 cm. beyond the further surface. This image must be considered as a virtual object (u_2 negative) for the refraction at the second surface. In this case ${}_g\mu_a=\frac{1}{1\cdot5}$

$$\frac{\mu}{v_2}-\frac{1}{u_2}=\frac{\mu-1}{R_2}$$

$$\frac{1}{1\cdot5v_2}-\frac{1}{-5}=\frac{\frac{1}{1\cdot5}-1}{5}$$

$$\frac{1}{1\cdot5v_2}=\frac{-0\cdot5}{5\times1\cdot5}-\frac{1}{5}$$

$$\frac{1}{v_2}=\frac{-0\cdot5-1\cdot5}{5}$$

$$\underline{v_2=-2\cdot5 \text{ cm.}}$$

$$\frac{\mu}{v_2}+\frac{1}{u_2}=\frac{\mu-1}{R_2}$$

$$\frac{1}{1\cdot5v_2}+\frac{1}{-5}=\frac{\frac{1}{1\cdot5}-1}{-5}$$

{$R_2=-5$ cm. Surface concave to the light}

$$\frac{1}{1\cdot5v_2}=\frac{-0\cdot5}{-5\times1\cdot5}+\frac{1}{5}$$

$$\frac{1}{v_2}=\frac{0\cdot5+1\cdot5}{5}$$

$$\underline{v_2=2\cdot5 \text{ cm.}}$$

The principal focus of the sphere is therefore 2·5 cm. beyond the second surface.

121.

A symmetrical concave lens made of glass of refractive index 1·55 has a focal length of 1 metre. What is the radius of curvature of each of its surfaces?

I

R_1 will be positive and R_2 negative whichever way the light is travelling.

So let

$$R_2 = -R_1$$

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\therefore \quad \frac{1}{100} = 0{\cdot}55\left(\frac{1}{R_1} - \frac{1}{-R_1}\right)$$

$$= \frac{0{\cdot}55 \times 2}{R_1}$$

$$R_1 = 100 \times 0{\cdot}55 \times 2$$

$$= \underline{110 \text{ cm.}}$$

II

Since both faces are concave, R_1 and R_2 are both negative.

$$\text{Let} \quad R_1 = R_2 = -x$$

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\therefore \quad \frac{1}{-100} = 0{\cdot}55\left(\frac{-2}{x}\right)$$

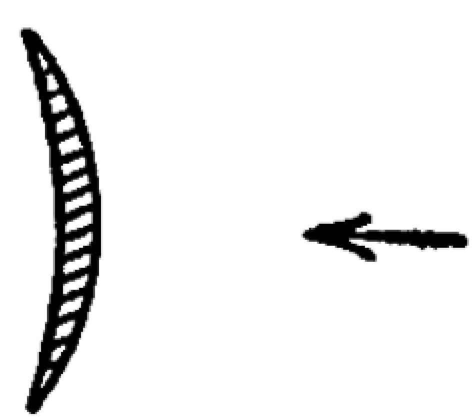
$$x = 2 \times 100 \times 0{\cdot}55$$

$$= \underline{110 \text{ cm.}}$$

Each surface has a radius of curvature of 110 cm.

122.

A convergent meniscus lens made of glass of refractive index 1·52 has surfaces whose radii of curvature are 20 cm. and 30 cm. respectively. What is its focal length?



As the lens is convergent, its convex surface must have the smaller radius of curvature.

Suppose the light is coming from the right.

I

Then the direction of the arrow in the diagram is the negative direction and both R_1 and R_2 are negative.

$R_1 = -20$ cm.

$R_2 = -30$ cm.

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= (1{\cdot}52 - 1)\left(\frac{1}{-20} - \frac{1}{-30}\right)$$

$$= 0{\cdot}52\left(\frac{-3+2}{60}\right)$$

$$= \frac{-0{\cdot}52}{60}$$

$$\underline{f = -115{\cdot}4 \text{ cm.}}$$

II

Then R_1 is the radius of the convex surface and R_2 that of the concave.

$R_1 = 20$ cm.

$R_2 = -30$ cm.

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$= (1{\cdot}52 - 1)\left(\frac{1}{20} - \frac{1}{30}\right)$$

$$= 0{\cdot}52\left(\frac{3-2}{60}\right)$$

$$= \frac{0{\cdot}52}{60}$$

$$\underline{f = 115{\cdot}4 \text{ cm.}}$$

The same result is obtained if the light is supposed to come from the left.

123.

One surface of a convergent lens of 20 cm. focal length is convex and has a radius of curvature of 25 cm. If the refractive index of the glass of which the lens is composed is 1·50, what is the radius of curvature of the other surface?

(a) (b) (c)

The lens will be of one of the three forms shown. Suppose the light to be incident first on the 25-cm. radius surface.

I

Then

$R_1 = -25$ cm.

$f = -20$ cm.

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{-20} = 0{\cdot}5\left(\frac{1}{-25} - \frac{1}{R_2}\right)$$

$$-\frac{1}{10} = -\frac{1}{25} - \frac{1}{R_2}$$

$$\frac{1}{R_2} = \frac{1}{10} - \frac{1}{25}$$

$$= \frac{5-2}{50} = \frac{3}{50}$$

$$\underline{R_2 = +16{\cdot}7 \text{ cm.}}$$

The plus sign indicating that the surface is convex and the lens therefore of type (*a*).

II

Then

$R_1 = 25$ cm.

$f = 20$ cm.

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\frac{1}{20} = 0{\cdot}5\left(\frac{1}{25} + \frac{1}{R_2}\right)$$

$$\frac{1}{10} = \frac{1}{25} + \frac{1}{R_2}$$

$$\frac{1}{R_2} = \frac{1}{10} - \frac{1}{25}$$

$$= \frac{5-2}{50} = \frac{3}{50}$$

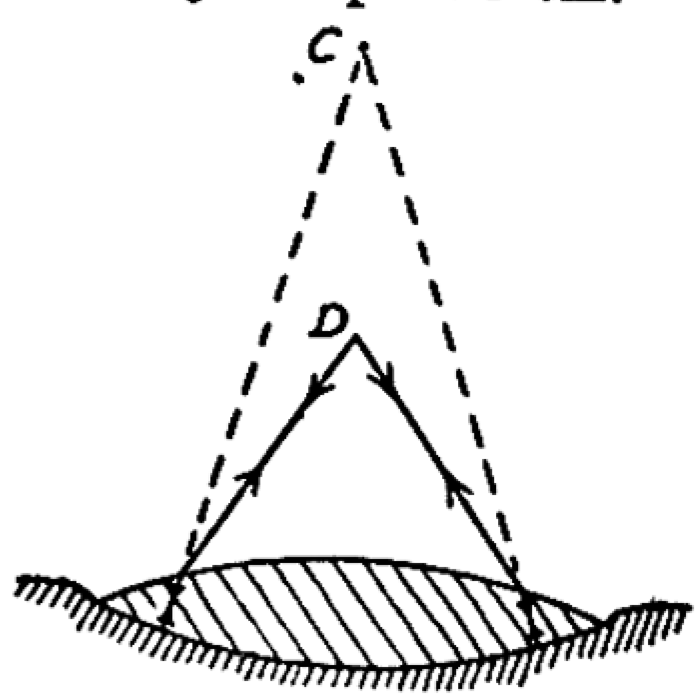
$$\underline{R_2 = +16{\cdot}7 \text{ cm.}}$$

The plus sign indicating that the surface is convex and the lens therefore of type (*a*).

124.

A symmetrical convex lens is floated on mercury and it is found that if a pin is held horizontally at a height of 25 cm. above the lens, the image of the pin formed by reflection from the lower surface of the lens shows no parallax with the pin itself. If the refractive index of the material of the lens is 1·5, what is the focal length of the lens?

Let C be the centre of curvature of the lower surface of the lens, and D the "apparent" centre as located by the pin 25 cm. above the lens. Since the reflected rays return along their incident path, they must meet the reflecting surface normally, and so within the lens must travel along radii of curvature. If the mercury were removed so that the rays passed undeviated through the surface, they would appear to come from C. C is therefore the position of the virtual image of D formed by rays passing through the lens.



Substituting in the lens equation:

I

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$$

we have:

(1) $$\frac{1}{f}=\frac{1}{R}-\frac{1}{25}$$

and from:

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\frac{1}{f}=(1{\cdot}5-1)\left(-\frac{2}{R}\right)$$

$\therefore\ f=-R$

from equation (1):

$$\frac{2}{f}=\frac{-1}{25}$$

$\therefore\ \underline{f=-50 \text{ cm.}}$

II

$$\frac{1}{f}=\frac{1}{v}+\frac{1}{u}$$

we have:

$$\frac{1}{f}=-\frac{1}{R}+\frac{1}{25} \qquad (1)$$

and from:

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$$

$$\frac{1}{f}=(1{\cdot}5-1)\left(\frac{2}{R}\right)$$

$\therefore f=R$

from equation (1):

$$\frac{2}{f}=\frac{1}{25}$$

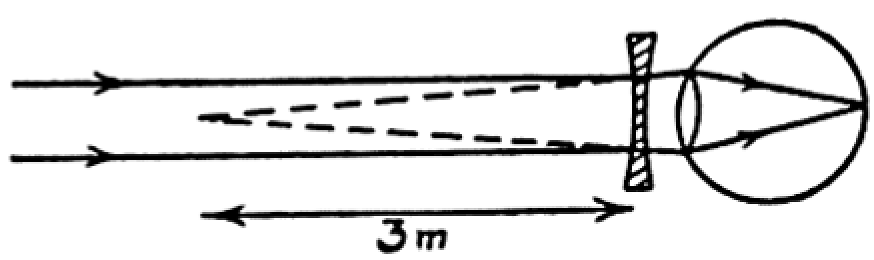
$\therefore \underline{f=50 \text{ cm.}}$

125.

A man, suffering from short sight, is unable to see objects distinctly at a distance greater than 3 metres. Of what power must spectacle lenses be which will enable him to view distant objects distinctly?

Rays from infinity must, after refraction by the lens, appear to come from 3 metres distance. That is, parallel rays falling on the lens must diverge from a virtual focus 3 metres from the lens. The lens must therefore be concave and of 3 metres focal length, its power being $-\frac{1}{3}$ dioptre.

Notice that for *short sight* a *concave* spectacle lens is needed of focal length equal to the distance of the far point.



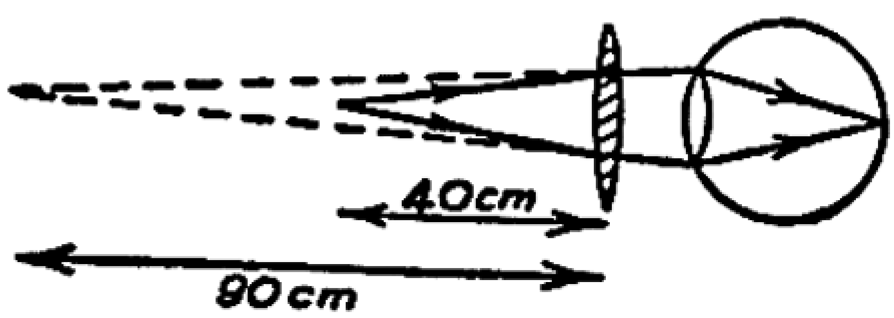
126.

What spectacles would be needed by a long-sighted person whose near point is at 90 cm. in order that he may be able to read print at a distance of 40 cm.?

Rays from a point 40 cm. away must appear to come from a point 90 cm. away after passing through the lens. That is, an object at a distance of 40 cm. must have a virtual image at a distance of 90 cm.

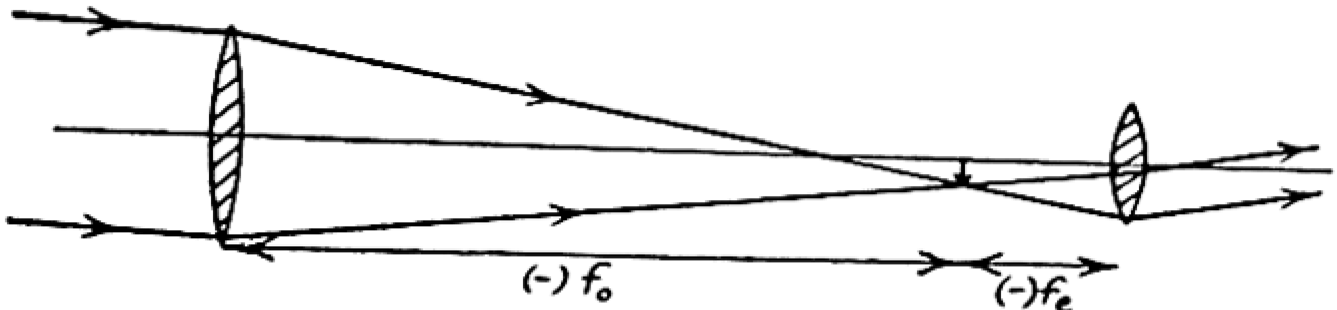
I	II
$u=40$ cm.	$u=40$ cm.
$v=90$ cm.	$v=-90$ cm.
$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$	$\frac{1}{f}=\frac{1}{v}+\frac{1}{u}$
$\frac{1}{f}=\frac{1}{90}-\frac{1}{40}$	$\frac{1}{f}=\frac{1}{-90}+\frac{1}{40}$
$=\frac{4-9}{360}$	$=\frac{-4+9}{360}$
$=\frac{-5}{360}$	$=\frac{5}{360}$
$\underline{f=-72\text{ cm.}}$	$\underline{f=72\text{ cm.}}$

For reading spectacles, the long-sighted person would need <u>convex lenses of focal length 72 cm.</u> (power$=\frac{1}{0{\cdot}72}=+1{\cdot}39$ dioptres).



127.

An astronomical telescope having a magnifying power of 8 consists of two thin lenses 18 in. apart. What are their focal lengths?

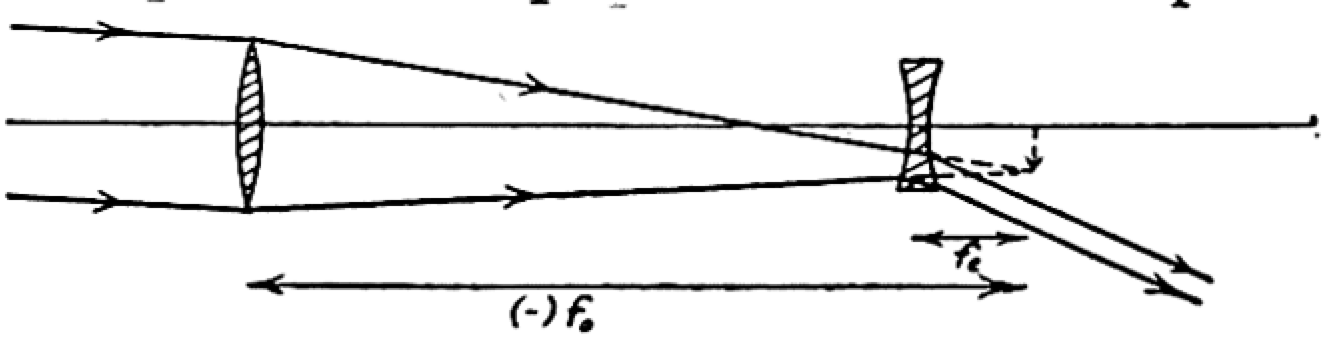


Calling the focal lengths of the objective and eyepiece f_o and f_e, we have:

I		II	
	$8=\frac{f_o}{f_e}$		$8=\frac{f_o}{f_e}$
and	$18=-(f_o+f_e)$	and	$18=f_o+f_e$
	$18=-(8f_e+f_e)$		$18=8f_e+f_e$
	$\underline{f_e=-2 \text{ in.}}$		$\underline{f_e=2 \text{ in.}}$
	$\underline{f_o=-16 \text{ in.}}$		$\underline{f_o=16 \text{ in.}}$

128.

Work the previous example for a Galilean telescope.



I

Since f_o is negative and f_e positive,

$$-8=\frac{f_o}{f_e}$$

and $18=(-f_o)-f_e$

$18=8f_e-f_e$

$\underline{f_e=2{\cdot}57 \text{ in.}}$

$\underline{f_o=-20{\cdot}6 \text{ in.}}$

II

Since f_o is positive and f_e negative,

$$-8=\frac{f_o}{f_e}$$

and $18=f_o-(-f_e)$

$18=-8f_e+f_e$

$\underline{f_e=-2{\cdot}57 \text{ in.}}$

$\underline{f_o=20{\cdot}6 \text{ in.}}$

129.

The focal lengths of the objective and eyepiece of a compound microscope are 1 cm. and 2 cm. respectively and the lenses are fixed 15 cm. apart. Find the magnifying power when the microscope is focussed so that the final image is formed at infinity.

If the final image is at infinity, the primary image must be at the principal focus of the eyepiece at a distance, therefore, of 13 cm. from the objective.

Applying the lens equation to the objective:

I

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$$

$$-\frac{1}{1}=-\frac{1}{13}-\frac{1}{u}$$

$$\frac{1}{u}=\frac{1}{1}-\frac{1}{13}$$

$$=\frac{12}{13}$$

$u=1\frac{1}{12}$ cm.

The magnification produced by the objective is therefore

$$\frac{13}{1\frac{1}{12}}=12$$

The magnification produced by the eyepiece used as a simple magnifier with the object at the principal focus is

$$\frac{25}{2}=12{\cdot}5$$

taking the least distance of distinct vision as 25 cm.

II

$$\frac{1}{f}=\frac{1}{v}+\frac{1}{u}$$

$$\frac{1}{1}=\frac{1}{13}+\frac{1}{u}$$

$$\frac{1}{u}=\frac{1}{1}-\frac{1}{13}$$

$$=\frac{12}{13}$$

$u=1\frac{1}{12}$ cm.

The magnification produced by the objective is therefore

$$\frac{13}{1\frac{1}{12}}=12$$

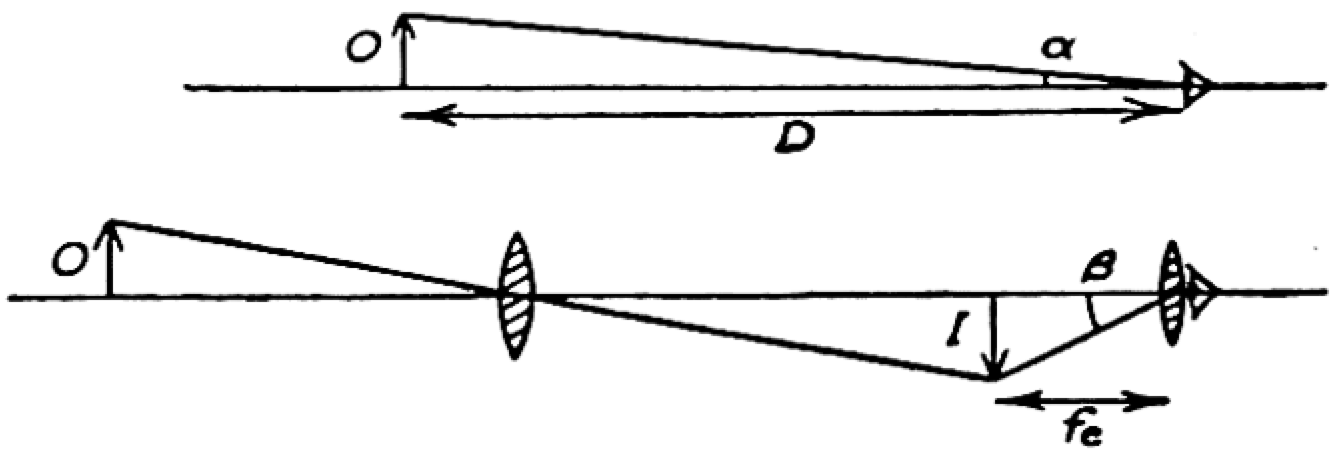
The magnification produced by the eyepiece used as a simple magnifier with the object at the principal focus is

$$\frac{25}{2}=12{\cdot}5$$

taking the least distance of distinct vision as 25 cm.

The total magnifying power is therefore $12\times 12{\cdot}5=150$.

The justification for regarding the total magnifying power as the product of the magnifications produced by the objective and eyepiece respectively is as follows:



The magnifying power of a compound microscope is defined as the ratio of the angle subtended at the eye by the image, to the angle which the object would subtend at the least distance of distinct vision. Referring to the diagram, this ratio is equal to $\frac{\beta}{a}$

If O and I are the respective heights of the object and primary image, then

$$a=\frac{O}{D}$$

$$\beta=\frac{I}{f_e}$$

where f_e is the focal length of the eyepiece, therefore

$$M=\frac{\beta}{a}$$

$$=\frac{I}{f_e}\times\frac{D}{O}$$

$$=\frac{I}{O}\times\frac{D}{f_e}$$

In this expression $\frac{I}{O}=$ magnification of primary image produced by the objective

and $\frac{D}{f_e}=$ magnification produced by eye-piece used as a simple microscope.

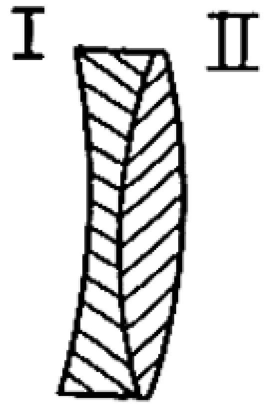
130.

Find the focal lengths of the two components of an achromatic telescope objective of focal length 1 metre if two specimens of glass are available for which the values of the refractive index for red and blue light are

	μ_r	μ_v
I Flint glass	1·630	1·650
II Crown glass	1·515	1·525

The dispersive power of a medium, in terms of μ for red, blue and yellow, is given by

$$\omega=\frac{\mu_v-\mu_r}{\mu_y-1}$$



where μ_y may be taken as $\frac{1}{2}(\mu_r+\mu_v)$

so

$$\omega_1=\frac{0{\cdot}020}{0{\cdot}64}=0{\cdot}03125$$

$$\omega_2=\frac{0{\cdot}010}{0{\cdot}52}=0{\cdot}01923$$

Two equations may now be written down, the first, the condition for achromatism:

$$\frac{f_1}{f_2}=-\frac{\omega_1}{\omega_2}=-\frac{\cdot03125}{\cdot01923}=-1{\cdot}625$$

$$f_2=\frac{-f_1}{1{\cdot}625}$$

and the second, using the fact that the focal length of the combination is to be 100 cm.

I

$$\frac{1}{f_1}+\frac{1}{f_2}=-\frac{1}{100}$$

Substitute for f_2 in this

$$\frac{1}{f_1}-\frac{1{\cdot}625}{f_1}=-\frac{1}{100}$$

$$\frac{-0{\cdot}625}{f_1}=-\frac{1}{100}$$

$$\underline{f_1=62{\cdot}5\text{ cm.}}$$

(divergent, flint glass)

and
$$f_2=\frac{-62{\cdot}5}{1{\cdot}625}$$
$$=\underline{-38{\cdot}5\text{ cm.}}$$

(convergent, crown glass)

II

$$\frac{1}{f_1}+\frac{1}{f_2}=\frac{1}{100}$$

Substitute for f_2 in this

$$\frac{1}{f_1}-\frac{1{\cdot}625}{f_1}=\frac{1}{100}$$

$$\frac{-0{\cdot}625}{f_1}=\frac{1}{100}$$

$$\underline{f_1=-62{\cdot}5\text{ cm.}}$$

(divergent, flint glass)

and
$$f_2=\frac{-62{\cdot}5}{-1{\cdot}625}$$
$$=\underline{38{\cdot}5\text{ cm.}}$$

(convergent, crown glass)

131.

A ROPE weighing 50 gm. per metre length is stretched, at a tension of 25 kgm. weight, between two points 30 metres apart. If the rope is plucked at one end, how long will it take for the resulting disturbance to reach the other end?

The velocity of propagation of a disturbance along a stretched string is given by:

$$v=\sqrt{\frac{T}{m}}$$

T being the tension in the string and m the mass per unit length. From the data,

$$\begin{cases} T=25{,}000\times 981 \text{ dynes} \\ m=\dfrac{50}{100} \text{ gm. per cm.} \end{cases}$$

$$\therefore \quad v=\sqrt{\frac{25{,}000\times 981}{0\cdot 5}} \text{ cm. per sec.}$$

$$=500\sqrt{98\cdot 1\times 2}$$
$$=500\times 14\cdot 01$$
$$=7005 \text{ cm. per sec.}$$

$\therefore$ Time taken for disturbance to travel 30 metres

$$=\frac{30\times 100}{7005} \text{ sec.}$$
$$=\underline{0\cdot 43 \text{ sec.}}$$

132.

A steel wire, 1 mm. in diameter and 1 metre long, hangs vertically from a rigid support with a mass of 10 kgm. attached to its lower end. What is the fundamental frequency of transverse vibration of the wire? Density of steel=7·8 gm. per c.c.

In the expression for the fundamental frequency of a stretched string

$$n=\frac{1}{2L}\sqrt{\frac{T}{m}}$$

$$\text{we are given}\begin{cases} L=100 \text{ cm.} \\ T=10 \text{ kgm. wt.}=10{,}000\times 981 \text{ dynes} \\ m, \text{ the mass per unit length} \end{cases}$$

$$=7{\cdot}8\times(\text{volume per unit length})$$
$$=7{\cdot}8\times\pi\times 0{\cdot}05^2$$
$$=0{\cdot}06125 \text{ gm. per cm.}$$

$$\therefore \qquad n=\frac{1}{2\times 100}\sqrt{\frac{9{\cdot}81\times 10^6}{0{\cdot}06125}}$$

$$=\frac{1000}{200}\sqrt{\frac{981}{6{\cdot}125}}$$

$$=5\times 12{\cdot}66$$

$$=\underline{63{\cdot}3}$$

133.

The movable bridge of a sonometer is adjusted so that 4 beats per second are heard when the string is sounded simultaneously with a tuning fork. The length of the vibrating portion of the string is found to be 60 cm. When the bridge is moved so as to lengthen the string by 1 cm., 4 beats per second are again obtained. What is the frequency of the fork?

Since the 60 and 61 cm. lengths of string both give 4 beats per second, the former must give a note which is sharp and the latter a note which is flat of that emitted by the fork.

The number of beats per second is equal to the difference between the frequencies, and therefore if n is the frequency of the fork, $(n+4)$ and $(n-4)$ are the frequencies of the string when it is 60 and 61 cm. long respectively.

If the tension is constant, the frequency of a string is inversely proportional to its length.

$$\therefore \qquad \frac{n+4}{n-4}=\frac{61}{60}$$

$$61n-244=60n+240$$

$$\therefore \qquad \underline{n=484}$$

134.

The four strings of a violin are tuned to the notes E, A, D and G respectively, the E being the highest in pitch. Assuming the strings to be made of the same material and the E string to

have a diameter of 0·4 mm., what must be the diameters of the other strings if all four strings are at the same tension?

The interval between each pair of adjacent strings is a fifth, giving a frequency of 3/2.

From the expression for the frequency of a vibrating string

$$n=\frac{1}{2L}\sqrt{\frac{T}{m}}$$

we may write
$$\frac{n_1}{n_2}=\sqrt{\frac{m_2}{m_1}}$$

if L and T are constant, and since m, the mass per unit length, is proportional to the radius squared

$$\frac{n_1}{n_2}=\frac{r_2}{r_1}$$

and therefore
$$\frac{3}{2}=\frac{r_A}{r_E}=\frac{d_A}{d_E}=\frac{d_A}{0{\cdot}4}$$

$$d_A=0{\cdot}6 \text{ mm.}$$

Similarly
$$d_D=\frac{3}{2}\times 0{\cdot}6=0{\cdot}9 \text{ mm.}$$

and
$$d_G=\frac{3}{2}\times 0{\cdot}9=1{\cdot}35 \text{ mm.}$$

The diameters of the A, D and G strings are 0·6 mm., 0·9 mm. and 1·35 mm. respectively.

135.

A steel wire 0·8 mm. in diameter is fixed to a rigid support at one end and is wrapped round a cylindrical tuning peg 5 mm. in diameter at the other, the length of wire between the peg and the support being 60 cm. Initially the wire is straight, under negligible tension. What will be the frequency of the wire if it is tightened by giving the peg a quarter of a turn?

Density of steel$=7{\cdot}8$ gm. per c.c.

Young's modulus for steel$=2\times 10^{12}$ dynes per sq. cm.

We have first to determine the tension in the wire.

Let it be T dynes.

The *stress* in the wire is the tension per unit area of cross-section which equals

$$\frac{T}{\pi \times 0{\cdot}04^2} \text{ dynes per sq. cm.}$$

The extension due to turning the tuning peg

$$=(\tfrac{1}{4} \times \pi \times 0{\cdot}5) \text{ cm.}$$

The *strain*, which is the extension per unit length,

$$=\frac{\pi \times 0{\cdot}5}{4 \times 60}$$

$$\frac{\text{stress}}{\text{strain}}=\text{Young's modulus}$$

$$\therefore \quad \frac{T}{\pi \times 0{\cdot}0016} \times \frac{4 \times 60}{\pi \times 0{\cdot}5}=2 \times 10^{12}$$

$$T=\frac{2 \times 10^{12} \times \pi^2 \times 0{\cdot}0008}{240} \text{ dynes}$$

$$=0{\cdot}6582 \times 10^8 \text{ dynes}$$

Now, in the equation for the frequency of a stretched string

$$n=\frac{1}{2L}\sqrt{\frac{T}{m}}$$

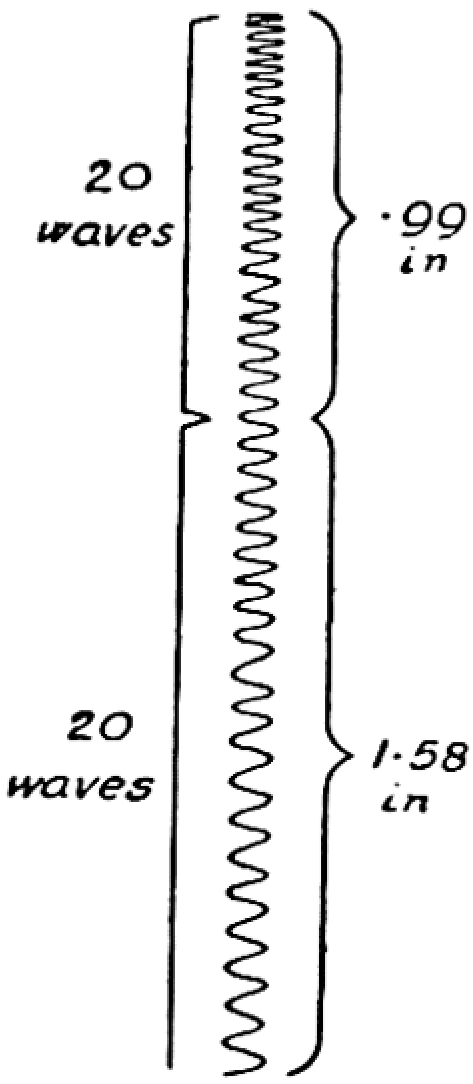
$$\left\{\begin{array}{l} L=60 \text{ cm.} \\ T=0{\cdot}6582 \times 10^8 \text{ dynes} \\ m=7{\cdot}8 \times \pi \times 0{\cdot}04^2 \text{ gm. per cm.} \end{array}\right.$$

$$\therefore \quad n=\frac{1}{2 \times 60}\sqrt{\frac{0{\cdot}6582 \times 10^8}{7{\cdot}8 \times 0{\cdot}0016 \times \pi}}$$

$$=\underline{341{\cdot}4}$$

136

A tuning fork of frequency 512 has a bristle attached to one of its prongs, bearing lightly on the smoked surface of a glass plate suspended vertically. On measuring the trace formed on the plate when the fork is sounded and the plate allowed to fall freely, it is found that two consecutive groups of 20 waves occupy 0·99 in. and 1·58 in. respectively. Deduce a value for g from these measurements.



Suppose, at the beginning of the 1st group of 20 waves, the plate was moving with velocity u ft. per second.

The 20 waves would occupy $\frac{20}{512}$ sec.

Therefore, from the equation

$$s=ut+\tfrac{1}{2}gt^2$$

$$\frac{0{\cdot}99}{12}=\frac{20u}{512}+\frac{g}{2}\left(\frac{20}{512}\right)^2 \text{ for the first group}$$

and $$\frac{0{\cdot}99+1{\cdot}58}{12}=\frac{40u}{512}+\frac{g}{2}\left(\frac{40}{512}\right)^2 \text{ for both groups}$$

u may be eliminated by multiplying the first equation by 2 and subtracting it from the second equation.

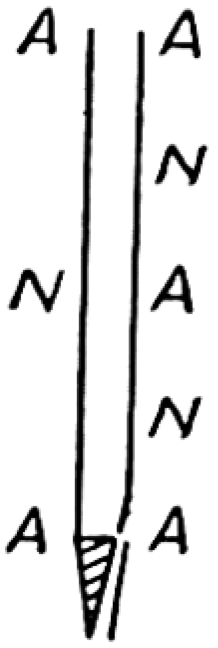
Thus: $$\frac{2{\cdot}57-1{\cdot}98}{12}=\frac{g}{2}\left[\left(\frac{40}{512}\right)^2-2\left(\frac{20}{512}\right)^2\right]$$

$$0{\cdot}0492=\frac{g}{2}\left(\frac{20}{512}\right)^2(4-2)=0{\cdot}001526g$$

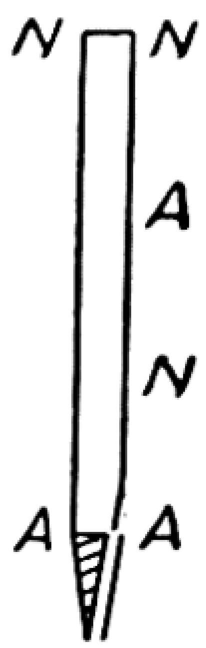
$$\therefore \qquad \underline{g=32{\cdot}3 \text{ ft. per sec. per sec.}}$$

137.

An organ pipe has a length of 2 metres. Neglecting end-corrections, find the frequency of its fundamental and first overtone when it is (*a*) open, and (*b*) closed, taking the velocity of sound in air as 34,000 cm. per sec.



a
open



b
closed

In each diagram the distribution of nodes and antinodes is shown; for the fundamental on the left, and for the first overtone on the right.

Bearing in mind that the distance between a node and the adjacent antinode is a quarter of the wavelength, we have for the wavelength of the fundamental and first overtone for (*a*) the open pipe:

$$\lambda_f=2\times 200 \text{ cm.}$$

$$\lambda_1=200 \text{ cm.}$$

whence the frequencies are:

$$n_f = \frac{V}{\lambda_f} = \frac{34000}{400} = \underline{85} \text{ (fundamental)}$$

$$n_1 = \frac{V}{\lambda_1} = \frac{34000}{200} = \underline{170} \text{ (first overtone)}$$

and for (*b*) the closed pipe:

$$\lambda_f = 4 \times 200 \text{ cm.}$$

$$\lambda_1 = \frac{4}{3} \times 200 \text{ cm.}$$

and
$$n_f = \frac{34000}{800} = \underline{42 \cdot 5} \text{ (fundamental)}$$

$$n_1 = \frac{34000 \times 3}{800} = \underline{127 \cdot 5} \text{ (first overtone)}$$

138.

A long glass tube is held vertically, dipping into water, while a tuning fork of frequency 512 is repeatedly struck and held over the open end. Strong resonance is obtained when the length of the tube above the surface of the water is 50 cm., and again at 84 cm., but at no intermediate point. Calculate the velocity of sound in air.

An air column in a tube closed at one end is capable of vibrating with any odd number of quarter-waves contained in its length, there being always a node at the closed end and an antinode at the open end.

The effective length of the air column is the actual length plus a constant end-correction. Let this end-correction be x cm.

Then our two resonating lengths are effectively $(50+x)$ and $(84+x)$ and these represent two consecutive odd multiples of $\frac{\lambda}{4}$.

The possible values of the resonant lengths are therefore:

$$\frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \frac{7\lambda}{4}, \ldots (2n-1)\frac{\lambda}{4}, (2n+1)\frac{\lambda}{4}, \text{etc.}$$

It is seen that any pair of consecutive values differ by $\frac{\lambda}{2}$.

$\therefore$
$$\frac{\lambda}{2} = (84+x) - (50+x)$$

$$= 34 \text{ cm.}$$

$\therefore$
$$\lambda = 68 \text{ cm.}$$

The velocity of sound in air is given by:

$$V = n\lambda$$
$$= 512 \times 68$$
$$= \underline{34{,}816 \text{ cm. per sec.}}$$

Since $\frac{\lambda}{4} = 17$ cm., and the first three possible resonant lengths are 17, 51 and 85 cm., it is apparent that the two tube lengths given represent the second and third resonances, and the end-correction is therefore equal to 1 cm.

139.

What change in temperature would cause the pitch of a wind instrument, originally at 5° C., to rise by a semi-tone? (Interval of a semi-tone $= \frac{17}{16}$.)

The velocity of sound in a gas is proportional to the square root of the absolute temperature.

Since the wavelength of the note emitted by a wind instrument is independent of the temperature, therefore the frequency also is proportional to the square root of the absolute temperature.

$$\therefore \qquad \frac{n_t}{n_5} = \sqrt{\frac{273+t}{278}}$$

If $t°$ is the temperature which would give a sharpening of the pitch by 1 semi-tone,

$$\frac{17}{16} = \sqrt{\frac{273+t}{278}}$$

$$\left(\frac{17}{16}\right)^2 = \frac{273+t}{278}$$

$$1{\cdot}129 = \frac{273+t}{278}$$

$$t = 40{\cdot}9° \text{ C.}$$

<u>A rise in temperature of 35·9° would cause a rise in pitch of 1 semi-tone.</u>

140.

In a Kundt's tube experiment, a brass rod 120 cm. long was used which had a fundamental longitudinal frequency of 1460.

The gas in the tube was carbon dioxide at 20° C., and the distance between the nodes, as shown by the dust heaps, was found to be 9·0 cm. Calculate the velocity of sound in brass and in carbon dioxide at 0° C.

As the rod was sounding its fundamental, it would have an antinode at each end and a node in the middle. The length of the rod would therefore be one-half the wavelength in brass.

$\therefore \qquad \lambda = 2 \times 120$ cm. in brass

and since $\quad V = n\lambda,$

$$V = (1460 \times 2 \times 120) \text{ cm. per sec.}$$
$$= 350{,}400 \text{ cm. per sec.}$$

The velocity of sound in brass=350,400 cm. per sec.

In the gas, the wavelength would be twice the distance between the nodes.

So $\qquad \lambda = 2 \times 9$ cm. in carbon dioxide

and $\qquad V = (1460 \times 2 \times 9)$

$$= 26{,}280 \text{ cm. per sec. at } 20^\circ \text{ C.}$$
$$= 26{,}280 \times \sqrt{\frac{273}{293}} \text{ cm. per sec. at } 0^\circ \text{ C.}$$
$$= 25{,}370 \text{ cm. per sec.}$$

The velocity of sound in carbon dioxide at 0° C.=25,370 cm. per sec.

141.

What is the frequency of the note heard by an observer standing close beside a railway track when an engine approaches him at 60 m.p.h., sounding its whistle emitting a note of frequency 512? Velocity of sound in air=1100 ft. per second.

If the engine were at rest, the wavelength of the sound waves emitted by the whistle would be $\frac{1100}{512}$ ft.

But, since the source of sound is following up the sound waves in the direction of the observer, at a speed of 60 m.p.h., or 88 ft. per sec., the centre of each wave will be $\frac{88}{512}$ ft. ahead of the

preceding one, and the wavelength will be

$$\left(\frac{1100}{512}-\frac{88}{512}\right)\text{ft.}=\frac{1012}{512}\text{ ft.}$$

The observer therefore receives waves of length $\frac{1012}{512}$ ft. travelling at their normal speed of 1100 ft. per sec., and the frequency of the note he hears is $\dfrac{1100}{\left(\frac{1012}{512}\right)}$ or $512\times\dfrac{1100}{1012}$

$$=\underline{556{\cdot}5}$$

The student may prefer to substitute in the general equation for the Doppler effect:

$$n'=n\left(\frac{V+u}{V-v}\right)$$

where V is the velocity of sound in air, and n' is the frequency received by an observer moving with velocity u towards a source which is approaching him with velocity v and emitting a note of frequency n.

$$n'=512\left(\frac{1100+0}{1100-88}\right)$$

$$=512\times\frac{1100}{1012}$$

$$=\underline{556{\cdot}5}$$

142.

A north pole of strength 20 c.g.s. units is situated 5 cm. away from a south pole of strength 30 units. Find the force which each pole exerts on the other.

The inverse square law of force between two magnetic poles is

$$F=\frac{m_1 m_2}{d^2}$$

In this question

$$m_1=20 \text{ units}$$
$$m_2=30 \text{ units}$$
$$d=5 \text{ cm.}$$

$$F=\frac{20\times 30}{5^2} \text{ dynes}$$
$$=\frac{600}{25} \text{ dynes}$$
$$=\underline{24 \text{ dynes}}$$

Since the poles are of opposite polarities, the force is one of attraction.

Therefore each pole attracts the other with a force of 24 dynes.

143.

Two equal magnetic poles, 15 cm. apart, repel each other with a force of 36 dynes. What is the strength of each pole?

Substituting in the equation

$$F=\frac{m_1 m_2}{d^2}$$

we have:

$$36=\frac{m\times m}{15^2}$$
$$m=\sqrt{36\times 15^2}$$
$$=90 \text{ c.g.s. units}$$

144.

Find the position of the neutral point for two north poles 10 cm. apart, of strengths 20 and 30 units respectively.

At the neutral point, where the magnetic intensity is zero, the field strength due to one pole must be equal and opposite to that due to the other. This can occur only on the straight line joining the two poles.

Let the neutral point P be at a distance x cm. from Pole A and y cm. from pole B.

Then field at P due to A$=\frac{20}{x^2}$ directed to the right

field at P due to B$=\frac{30}{y^2}$ directed to the left

For zero intensity at P

$$\frac{20}{x^2}=\frac{30}{y^2}$$

$$\frac{x^2}{y^2}=\frac{20}{30}$$

$$\frac{x}{y}=\sqrt{\frac{2}{3}}$$

$$=0{\cdot}816$$

Also

$$x+y=10$$

$$0{\cdot}816y+y=10$$

$$y=\frac{10}{1{\cdot}816}$$

$$=\underline{5{\cdot}51\text{ cm.}}$$

The neutral point is situated on the straight line joining the two poles at a distance of 5·51 cm. from the stronger pole.

The problem may also be solved by using only one variable and one equation. This solution involves a quadratic equation.

Letting x be the distance of the neutral point from A as before, its distance from B is $(10-x)$.

Then
$$\frac{20}{x^2}=\frac{30}{(10-x)^2}$$
$$=\frac{30}{100+x^2-20x}$$
$$2(100+x^2-20x)=3x^2$$
$$x^2+40x-200=0$$
$$\therefore \qquad x=\frac{-40\pm\sqrt{1600+800}}{2}$$
$$=-20\pm\sqrt{600}$$
$$=-20\pm24{\cdot}49$$
$$=\underline{4{\cdot}49} \text{ or } -44{\cdot}49 \text{ cm.}$$

The positive solution gives the same result as the first method. The negative solution applies to the case where the two poles are of opposite polarities.

145.

Find the magnetic intensity due to a short bar magnet of moment 500 c.g.s. units at a point 30 cm. from the centre of the magnet (*a*) on the axis of the magnet, (*b*) on a line through the centre at right angles to the axis.

By a "short" magnet is meant one for which l^2 is negligibly small in comparison with d^2, so that the approximate expressions for the magnetic intensity may be used:

(*a*)
$$H=\frac{2M}{d^3}$$
$$=\frac{2\times500}{30^3}$$
$$=\frac{1000}{27000}$$
$$=\underline{0{\cdot}037 \text{ dynes per unit pole (or oersted)}}$$

(b) The approximate expression for the intensity at a point on a line through the centre perpendicular to the axis is

$$H=\frac{M}{d^3}$$
$$=\frac{500}{30^3}$$
$$=\underline{0\cdot0185 \text{ oersted}}$$

146.

Work the previous example for a magnet 10 cm. long.

The exact expressions for the field strength must now be used.

(a)
$$H=\frac{2Md}{(d^2-l^2)^2}$$
$$=\frac{2\times500\times30}{(30^2-5^2)^2}$$
$$=\frac{30000}{(900-25)^2}$$
$$=\underline{0\cdot0392 \text{ oersted}}$$

(b)
$$H=\frac{M}{(d^2+l^2)^{\frac{3}{2}}}$$
$$=\frac{500}{(30^2+5^2)^{\frac{3}{2}}}$$
$$=\frac{500}{925^{\frac{3}{2}}}$$
$$\log H=\log 500-\left(\frac{3}{2}\times\log 925\right)$$
$$=2\cdot6990-\left(\frac{3}{2}\times2\cdot9661\right)$$
$$=\bar{2}\cdot2499$$
$$\therefore \qquad H=\underline{0\cdot0178 \text{ oersted}}$$

147.

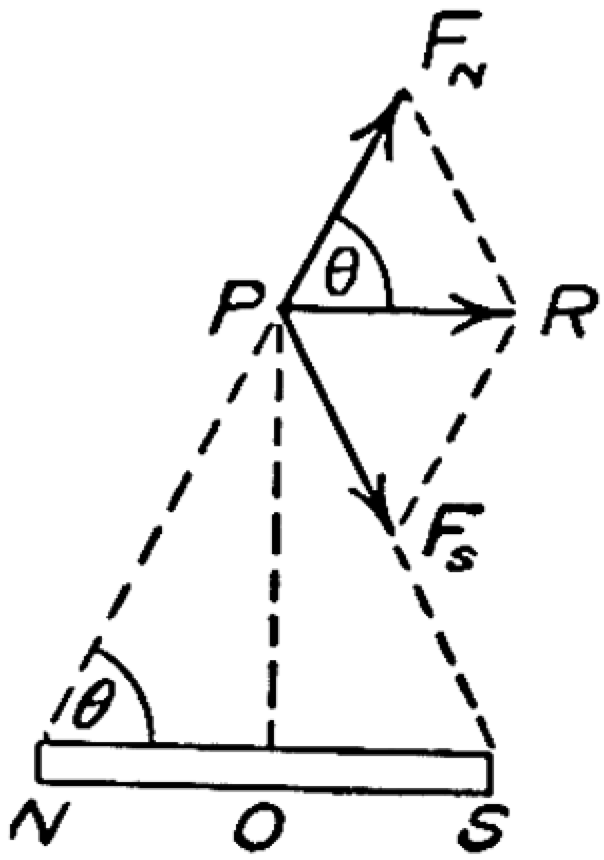
What is the value of the magnetic intensity due to a bar magnet of magnetic moment 600 c.g.s. units and length 10 cm. at a point on the perpendicular bisector of the magnet, 12 cm. from its centre?

Referring to the diagram,
Field at P due to N pole:

$$F_N=\frac{m}{\mathsf{NP}^2}\text{ in the direction NP}$$

Field at P due to S pole:

$$F_S=\frac{m}{\mathsf{SP}^2}\text{ in the direction PS}$$



These two field strengths are equal in magnitude, since NP=SP. Their resultant may be found by applying the parallelogram of forces, or by resolving F_N and F_S in a direction parallel to NS.

Thus $R=(F_N+F_S)\cos\theta$

$$=2F_N\times\frac{\mathsf{NO}}{\mathsf{NP}}$$

$$=\frac{2m\times\mathsf{NO}}{\mathsf{NP}^3}$$

$$=\frac{M}{(\sqrt{\mathsf{NO}^2+\mathsf{OP}^2})^3}$$

$$=\frac{600}{(\sqrt{5^2+12^2})^3}$$

$$=\frac{600}{13^3}$$

$$=\underline{0{\cdot}273\text{ oersted}}$$

The same result is obtained by substituting directly in the "broadside-on" formula:

$$R=\frac{M}{(d^2+l^2)^{\frac{3}{2}}}$$

in which $d=12$ cm.
and $l=5$ cm.

148.

A magnet, placed on one arm of a deflection magnetometer in the "end-on" position, with its centre 25 cm. from the centre of the needle, produces a deflection of 40°. If the distance between the poles of the magnet is 10 cm., what is its magnetic moment?

What deflection would be produced if the magnet were placed at an equal distance in the "broadside-on" position? (H=0·18 oersted.)

In the "end-on" position

$$\frac{2Md}{(d^2-l^2)^2}=\mathrm{H}\tan\theta$$

We are given
$$\begin{cases} d=25 \text{ cm.} \\ l=\frac{1}{2}\times 10 \text{ cm.} \\ \theta=40° \\ \mathrm{H}=0{\cdot}18 \text{ oersted} \end{cases}$$

$$\therefore \qquad \frac{2M\times 25}{(25^2-5^2)^2}=0{\cdot}18\times\tan 40°$$

$$=0{\cdot}18\times 0{\cdot}839$$

$$M=\frac{(625-25)^2\times 0{\cdot}18\times 0{\cdot}839}{50}$$

$$=\frac{360000\times 0{\cdot}18\times 0{\cdot}839}{50}$$

$$=\underline{1087 \text{ dyne cm. per unit field}}$$

(The last figure is unreliable, owing to the experimental difficulty of reading the deflection to less than ½°.)

In the "broadside-on" position

$$\frac{M}{(d^2+l^2)^{\frac{3}{2}}}=\mathrm{H}\tan\theta$$

$$\tan\theta=\frac{1087}{0{\cdot}18\times(625+25)^{\frac{3}{2}}}$$

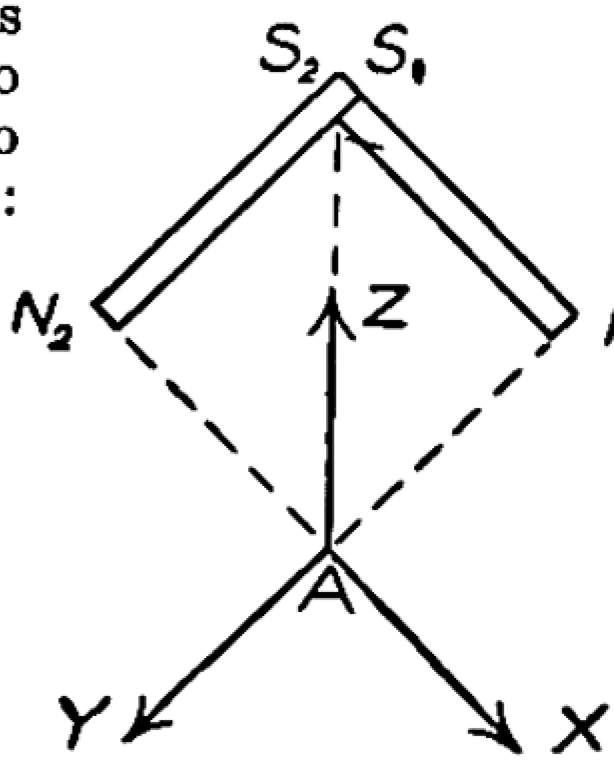
$$=0{\cdot}365$$

$$\underline{\theta=20° \text{ to the nearest } \tfrac{1}{2}°}$$

149.

Two similar bar magnets, 10 cm. long, each having poles of strength 45 c.g.s. units, are laid on a horizontal table with their S. poles in contact and their axes at right angles, the magnetic meridian being along the bisector of the right angle. Assuming the poles to be situated at the ends of the magnets, find the magnetic intensity at the opposite corner of the square of which the magnets form two sides. **H** may be taken as equal to 0·18 oersted.

Referring to the diagram, the field strength at A due to the magnets is the resultant of X, the field due to N_2, Y, due to N_1 and Z, due to S_1 and S_2. By the inverse square law:



$$X=\frac{45}{10^2}=0{\cdot}45 \text{ oersted}$$

$$Y=\frac{45}{10^2}=0{\cdot}45 \text{ oersted}$$

$$Z=\frac{2\times 45}{SA^2}$$

$$=\frac{90}{10^2+10^2}=0{\cdot}45 \text{ oersted}$$

By the triangle of forces, the resultant of X and Y, acting along the diagonal in opposition to Z, is equal to

$$\sqrt{X^2+Y^2}=\sqrt{2\times 0{\cdot}45^2}=0{\cdot}636 \text{ oersted}$$

The resultant intensity due to both magnets therefore

$$=0{\cdot}636-Z=0{\cdot}186 \text{ oersted}$$

in the direction SA.

According as whether **H** is in the direction SA or AS, the horizontal magnetic intensity at A due to the combined action of the earth's field and the field due to the magnets is

$$0{\cdot}186+0{\cdot}18=\underline{0{\cdot}366 \text{ oersted}}$$

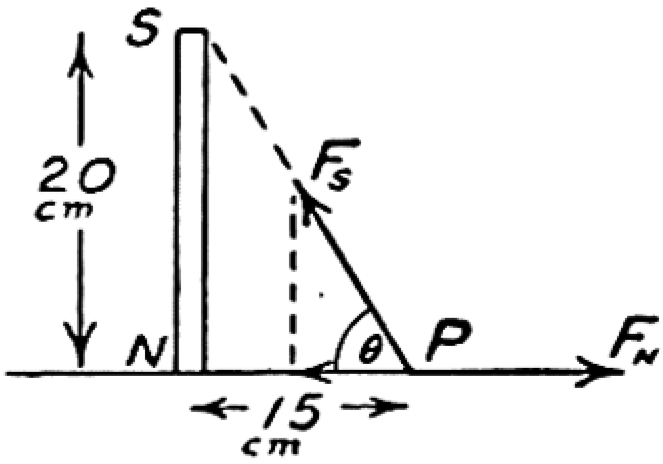
or

$$0{\cdot}186-0{\cdot}18=\underline{0{\cdot}006 \text{ oersted}}$$

150.

A bar magnet 20 cm. long is fixed vertically with its lower end resting on a horizontal sheet of paper. By means of a small compass needle, lines of force are plotted on the paper and a

neutral point is located 15 cm. from the magnet. If the earth's horizontal field strength is 0·20 oersted, find the moment of the magnet, assuming its poles to be situated at its ends.



Suppose that each pole is of strength m and that the lower pole is a north pole.

Then the horizontal field due to the lower pole at the neutral point P is

$$F_N=\frac{m}{15^2}$$

And the field due to the upper pole at P is

$$F_S=\frac{m}{\mathrm{SP}^2}=\frac{m}{20^2+15^2}$$

in the direction PS.

The horizontal component of this is

$$\frac{m}{625}\cdot\cos\theta$$
$$=\frac{m}{625}\cdot\frac{\mathrm{NP}}{\mathrm{SP}}$$
$$=\frac{m}{625}\cdot\frac{15}{\sqrt{625}}$$
$$=\frac{15m}{625\times 25}$$
$$=\frac{3m}{3125}$$

The resultant horizontal intensity at P due to both poles is therefore

$$\frac{m}{15^2}-\frac{3m}{3125}$$
$$=m\left(\frac{1}{225}-\frac{3}{3125}\right)$$
$$=m(0{\cdot}004444-0{\cdot}000960)$$
$$=0{\cdot}003484m$$

Since at the neutral point this field must be equal and opposite to the horizontal component of the earth's field,

$$\therefore \qquad 0{\cdot}003438m = 0{\cdot}20$$

$$m = 57{\cdot}4 \text{ c.g.s. units}$$

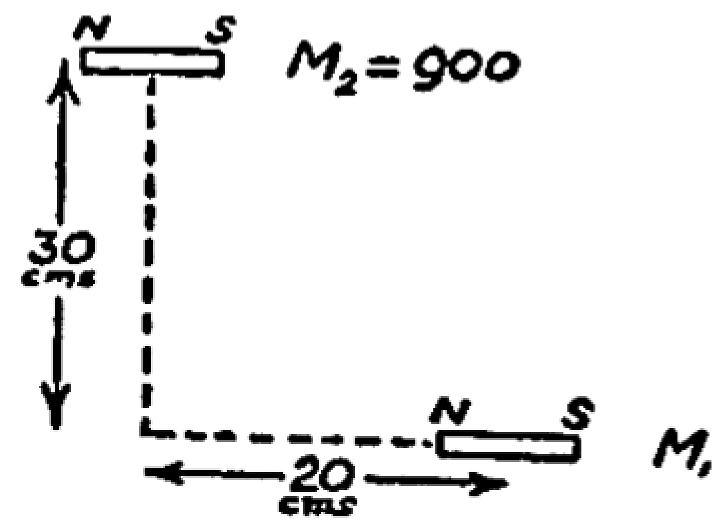
and the magnetic moment

$$M = 57{\cdot}4 \times 20$$

$$= \underline{1148 \text{ dyne cm. per unit field}}$$

151.

A compass needle is deflected by a short magnet having its axis east and west and its centre 20 cm. due east of the needle. The deflection is then reduced to zero by another magnet of moment 900 c.g.s. units, placed with its axis east and west and its centre 30 cm. due north of the needle. What is the magnetic moment of the first magnet?



Field at needle due to first magnet in the "end-on" position

$$= \frac{2M_1}{20^3}$$

Field at needle due to second magnet in the "broadside-on" position

$$= \frac{900}{30^3}$$

Since the deflection produced by the first magnet is neutralised by the second, the fields due to the two magnets must be equal and opposite.

$$\therefore \qquad \frac{2M_1}{20^3} = \frac{900}{30^3}$$

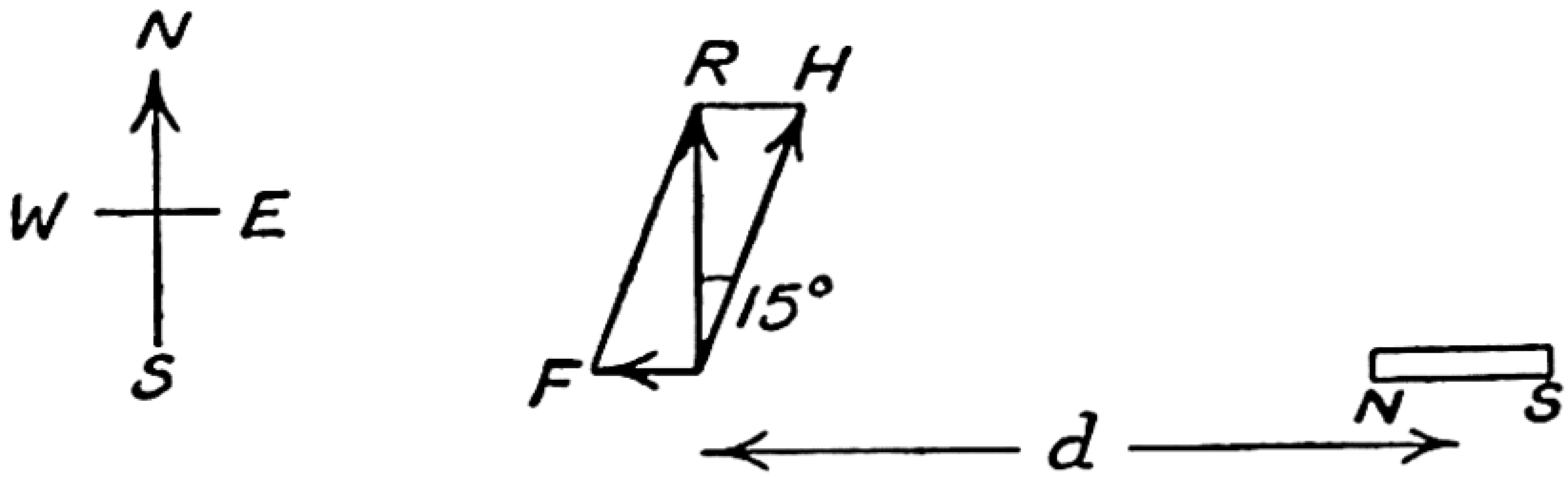
$$\therefore \qquad M_1 = \frac{8}{27} \times \frac{900}{2}$$

$$= \underline{133{\cdot}3 \text{ dyne cm. per unit field}}$$

152.

At a place where the value of **H** is 0·20 oersted and the magnetic declination is 15°, the axis of a compass needle is made to lie in the geographical meridian by placing a short magnet of

moment 500 c.g.s. units due east of the needle, with its axis east and west. What is the distance between the centres of the needle and the magnet?



The compass needle sets along the direction of the resultant of **H** and F (the field due to the magnet). F must therefore be directed towards the west, as shown in the diagram, the resultant R being at right angles to it, directed towards the north.

The parallelogram of forces is therefore divided by the vector R into two similar right-angled triangles, from which it may be seen that

$$\sin 15° = \frac{F}{\mathbf{H}}$$

$$F = \mathbf{H} \sin 15°$$

$$= 0{\cdot}20 \times 0{\cdot}259$$

$$= 0{\cdot}0518 \text{ oersted}$$

A short bar magnet of moment M produces a field of strength $\frac{2M}{d^3}$ at a point on its axis distant d from its centre.

$$\therefore \qquad 0{\cdot}0518 = \frac{2 \times 500}{d^3}$$

$$d^3 = \frac{1000}{0{\cdot}0518}$$

$$\log d = \frac{(3 - \bar{2}{\cdot}7143)}{3}$$

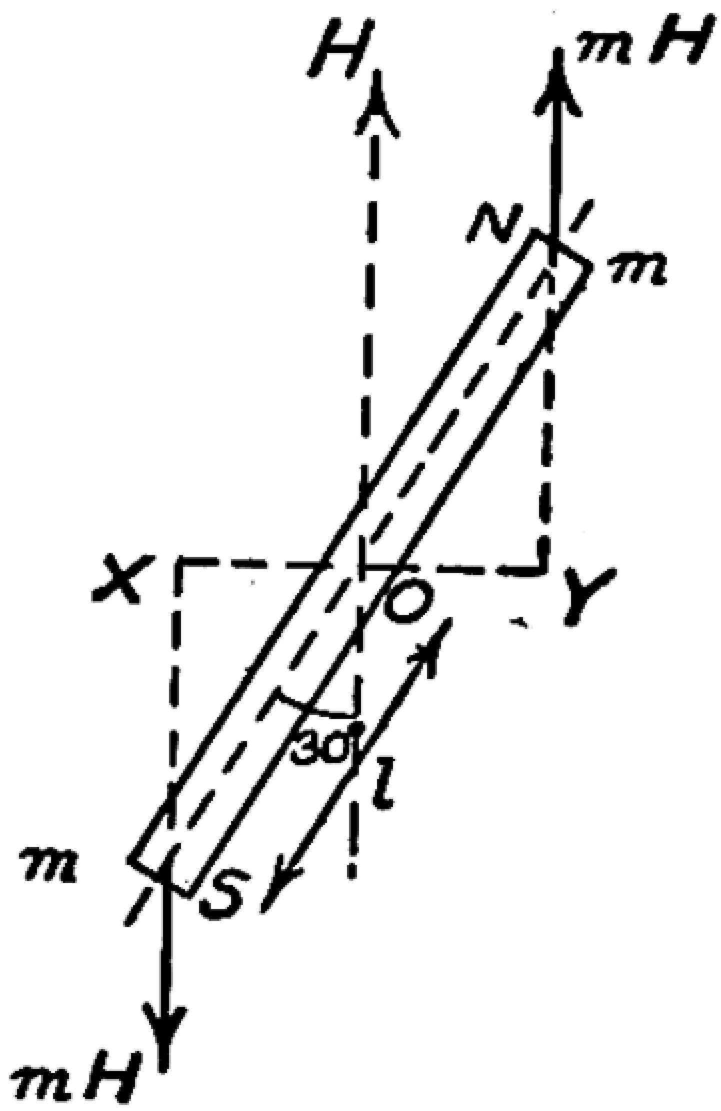
$$= 1{\cdot}4286$$

$$= \log 26{\cdot}8$$

$$\therefore \qquad \underline{d = 26{\cdot}8 \text{ cm.}}$$

153.

What couple would be required to hold a magnet of moment 500 c.g.s. units in a horizontal plane with its axis at an angle of 30° to the magnetic meridian if H=0·18 oersted?



In the diagram, let m be the pole strength of the magnet and $2l$ its length.

Then $2lm = \mathrm{M} = 500$ c.g.s. units

The force acting on each pole is $m\mathrm{H}$, and as the two forces are equal, parallel and oppositely directed, they constitute a couple of moment.

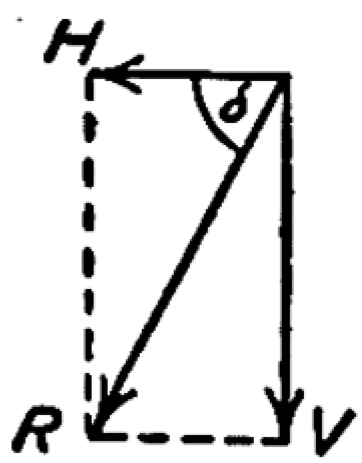
$$\begin{aligned} & m\mathrm{H} \times \mathrm{XY} \\ &= m\mathrm{H} \times 2\mathrm{OY} \\ &= m\mathrm{H} \times 2l \sin 30° \\ &= 2lm\mathrm{H} \sin 30° \\ &= M\mathrm{H} \sin 30° \\ &= 500 \times 0{\cdot}18 \times 0{\cdot}5 \text{ dyne cm.} \\ &= 45 \text{ dyne cm.} \end{aligned}$$

We have here found the couple tending to turn the magnet towards the meridian. The couple required to hold it in position must be equal and opposite, i.e. it must be a couple (clockwise in the diagram) of moment 45 dyne cm.

154.

What is the strength of the earth's vertical field at a place where the angle of dip is 67° and the earth's horizontal field strength is 0·18 dyne per unit pole?

In the diagram, **H** and **V** represent the horizontal and vertical components of the earth's total field **R**, inclined at δ to the horizontal, δ being the angle of dip.

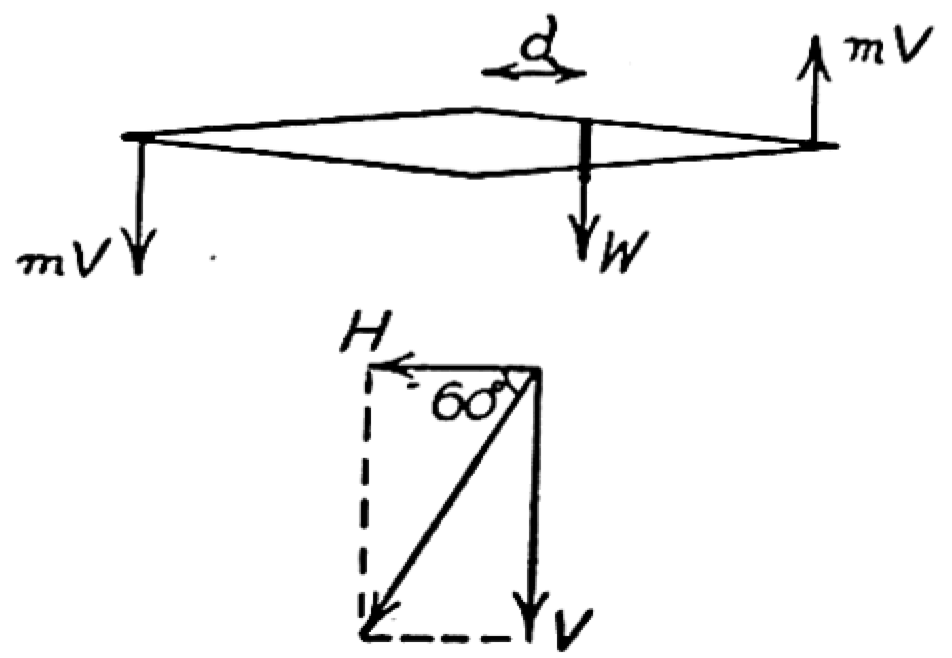


Therefore $\dfrac{\mathrm{V}}{\mathrm{H}} = \tan\delta$

$$\begin{aligned} \mathrm{V} &= \mathrm{H}\tan\delta \\ &= 0{\cdot}18 \times \tan 67° \\ &= 0{\cdot}18 \times 2{\cdot}356 \\ &= 0{\cdot}424 \text{ dyne per unit pole (or oersted)} \end{aligned}$$

155.

A dip needle of magnetic moment 300 c.g.s. units carries a 50-mgm. rider which is adjusted until the needle, when swinging in the meridian, sets horizontally at a place where **H** is equal to 0·20 oersted and the angle of dip is 60°. What is the distance measured along the axis of the needle between the axle and the rider?



For equilibrium, the moment of the couple due to magnetic forces must equal the moment of the weight of the rider about the axle.

I.e. $$MV = Wd$$

$$\therefore \quad MH \tan 60° = Wd$$

$$300 \times 0{\cdot}20 \times 1{\cdot}732 = 0{\cdot}05 \times 981 \times d$$

$$\underline{d = 2{\cdot}12 \text{ cm.}}$$

156.

A short compass needle makes 20 vibrations per min. in the earth's magnetic field. A bar magnet is placed in such a position that the time of swing of the needle becomes infinitely great. How many vibrations per min. will be made by the needle if the magnet is now reversed end for end?

The expression for the period of a vibrating magnet:

$$T = 2\pi\sqrt{\frac{I}{MH}}$$

may be written in terms of the frequency:

$$n = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{MH}{I}}$$

whence it follows that, for a particular magnet, vibrating successively in two fields of strengths H_1 and H_2,

$$\frac{n_1}{n_2} = \sqrt{\frac{H_1}{H_2}}$$

If the time of swing becomes infinite, H must be zero—that is, the needle is at a neutral point. At such a point the field due

to the earth (H) must be equal and opposite to that due to the magnet (−H), the total field being H−H=0.

On reversing the magnet, its field changes sign and becomes +H, the total field now being H+H=2H.

So, substituting in the above equation, in which H_1=H; H_2=2H, we have

$$\frac{20}{60n_2}=\sqrt{\frac{\mathrm{H}}{2\mathrm{H}}}$$

$$n_2=\frac{\sqrt{2}}{3}\text{ per sec.}$$

and the number of vibrations per min.

$$=60\times\frac{\sqrt{2}}{3}$$

$$=\underline{28{\cdot}3}$$

157.

A small compass needle makes 20 oscillations in 25 sec. in the earth's magnetic field. When placed due north of a bar magnet having its axis in the magnetic meridian, 20 oscillations occupy 12 sec. How many oscillations will be made in a minute if the magnet is reversed end for end?

From the expression $T=2\pi\sqrt{\frac{I}{MH}}$ for the period of an oscillating magnet or needle we may write $T\propto\frac{1}{\sqrt{H}}$ if we are dealing with a particular needle for which I and M are constant. If the needle is oscillating successively in two fields, then

$$\frac{T_1}{T_2}=\sqrt{\frac{H_2}{H_1}}\text{ or }\frac{T_1{}^2}{T_2{}^2}=\frac{H_2}{H_1}$$

In this problem H_1=H, the earth's horizontal field

and $$T_1=\frac{25}{20}\text{ sec.}$$

$$H_2=\mathrm{H}+F$$

F being the field due to the magnet

$$T_2=\frac{12}{20}\text{ sec.}$$

H$+F$ has been written rather than H$-F$ because, from the data, it is seen that T_1 is greater than T_2 and therefore H_2 must be greater than H_1.

So
$$\left(\frac{\frac{25}{20}}{\frac{12}{20}}\right)^2=\frac{\mathrm{H}+F}{\mathrm{H}}\quad\left\{\begin{array}{l}\text{The two unknowns H and } F \text{ can-}\\ \text{both be found from one equation,}\\ \text{but all that is required for the}\\ \text{solution is the ratio } F/\mathrm{H}.\end{array}\right.$$

$$\frac{625}{144}=1+\frac{F}{\mathrm{H}}$$

$$\therefore\qquad \frac{625}{144}-1=\frac{F}{\mathrm{H}}$$

$$\frac{481}{144}=\frac{F}{\mathrm{H}}$$

In the third case $H_3=\mathrm{H}-F$ or $F-\mathrm{H}$ according as whether $\mathrm{H}>F$ or $F>\mathrm{H}$. From the above value for F/H it is seen that the second alternative, $H_3=F-\mathrm{H}$, must be chosen.

Hence
$$\left(\frac{T_1}{T_3}\right)^2=\frac{H_3}{H_1}$$

and therefore

$$\left(\frac{\frac{25}{20}}{T_3}\right)^2=\frac{F-\mathrm{H}}{\mathrm{H}}=\frac{F}{\mathrm{H}}-1$$

Substituting the value obtained for F/H gives

$$\left(\frac{\frac{25}{20}}{T_3}\right)^2=\frac{481}{144}-1$$

$$\frac{25}{16T_3^2}=\frac{481-144}{144}$$

$$=\frac{337}{144}$$

$$\therefore\qquad T_3=\sqrt{\frac{25\times 144}{16\times 337}}$$

$$=\frac{5\times 12}{4\times\sqrt{337}}$$

$$=0{\cdot}817 \text{ sec.}$$

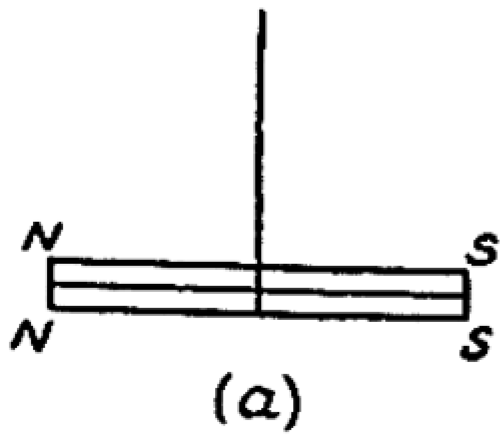
Therefore in 1 min. there would be $\frac{60}{0{\cdot}817}$ oscillations

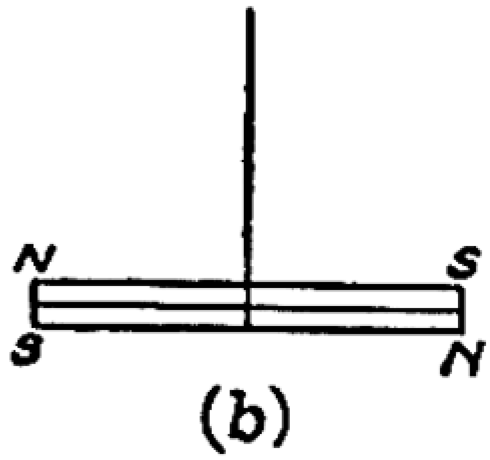
$$=\underline{73{\cdot}4 \text{ oscillations}}$$

Note that since $F>H$, the compass needle will have turned through 180°, its north end pointing south.

158.

Two bar magnets are fastened together alongside each other with similar poles in contact, and are allowed to execute small horizontal oscillations on the end of a silk fibre. It is found that 50 oscillations take place in 2 min. 32 sec. When one of the magnets is reversed end for end, 50 oscillations occupy 10 min. 15 sec. Find the ratio of the magnetic moments of the magnets.





Calling the moments of the magnets M_1 and M_2, the moment of the combination is (M_1+M_2) in the first case (a), and (M_1-M_2) in the second (b).

The periodic times are:

$$T_a=\frac{152}{50} \text{ sec.}$$

$$T_b=\frac{615}{50} \text{ sec.}$$

From the expression $$T=2\pi\sqrt{\frac{I}{MH}}$$

we may write $$\frac{T_a}{T_b}=\sqrt{\frac{I_a}{M_aH}\cdot\frac{M_bH}{I_b}}$$

and since the total moment of inertia is unaffected by reversing one of the magnets, therefore $I_a=I_b$

and $$\frac{T_a}{T_b}=\sqrt{\frac{M_b}{M_a}}=\sqrt{\frac{M_1-M_2}{M_1+M_2}}$$

or
$$\left(\frac{T_a}{T_b}\right)^2=\frac{M_1-M_2}{M_1+M_2}$$

$\therefore$
$$\left(\frac{152}{50}\cdot\frac{50}{615}\right)^2=\frac{M_1-M_2}{M_1+M_2}$$

$$\left(\frac{152}{615}\right)^2=\frac{M_1-M_2}{M_1+M_2}$$

$$0{\cdot}0611=\frac{M_1-M_2}{M_1+M_2}$$

$$\frac{M_1}{M_2}=\frac{1+0{\cdot}0611}{1-0{\cdot}0611}$$

$$=\underline{1{\cdot}128}$$

159.

A magnet whose moment of inertia is 650 gm. cm.^{2} and length 10 cm. makes 10 small oscillations in 2 min. when suspended freely so that it can swing in a horizontal plane. When placed with its axis east and west along one arm of a deflection magnetometer with its centre 30 cm. from the needle, it produces a deflection of 35°. Calculate the magnetic moment of the magnet and the horizontal intensity of the earth's magnetic field.

Vibration Experiment

The period of a magnet vibrating in the earth's horizontal field is

$$T=2\pi\sqrt{\frac{I}{M\mathrm{H}}}$$

In this problem $T=\dfrac{2\times 60}{10}=12$ seconds

and $I=650$ gm. cm.^{2}

$\therefore$
$$12=2\times 3{\cdot}142\sqrt{\frac{650}{M\mathrm{H}}}$$

whence
$$M\mathrm{H}=650\left(\frac{2\times 3{\cdot}142}{12}\right)^2$$

$$\underline{M\mathrm{H}=178{\cdot}3}$$

Deflection Experiment

In the "end-on" position

$$\frac{2Md}{(d^2-l^2)^2}=\text{H}\tan\theta$$

$$\left.\begin{array}{l} d=30 \text{ cm.} \\ l=5 \text{ cm.} \\ \theta=35° \end{array}\right\} \frac{2\times M\times 30}{(900-25)^2}=H\times\tan 35°$$

$$\therefore \qquad \frac{M}{\text{H}}=\frac{875^2\times 0{\cdot}7002}{60}$$

$$\underline{\frac{M}{\text{H}}=8932}$$

Combining the two results

$$M=\sqrt{M\text{H}\times\frac{M}{\text{H}}}=\sqrt{178{\cdot}3\times 8932}$$

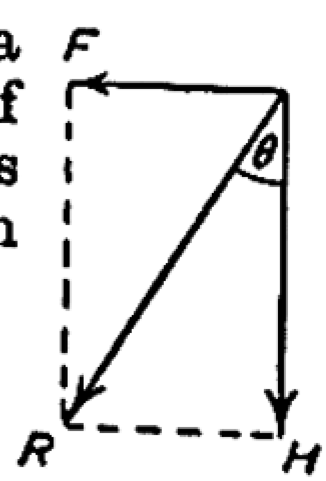
$$=\underline{1262 \text{ dyne cm. per unit field}}$$

$$\text{H}=\frac{M\text{H}}{M}=\frac{178{\cdot}3}{1262}$$

$$=\underline{0{\cdot}141 \text{ oersted}}$$

160.

The needle of a deflection magnetometer makes 10 oscillations of small amplitude in 30·4 sec. in the earth's magnetic field. When a bar magnet of length 10 cm. is placed in the "end-on" position, with its centre at a distance of 25 cm. from the needle, the time of 10 oscillations is 21·2 sec. Calculate the final angle of deflection of the needle and the moment of the magnet, H being 0·18 oersted.



The field in which the needle swings with a periodic time of 2·12 sec. is R, the resultant of F, the field due to the magnet, and H, the earth's horizontal field. Let the final angle of deflection be θ.

Then, since $\dfrac{T_1^2}{T_2^2}=\dfrac{H_2}{H_1}$

we have
$$\frac{3{\cdot}04^2}{2{\cdot}12^2}=\frac{R}{0{\cdot}18}=\frac{1}{\cos\theta}$$

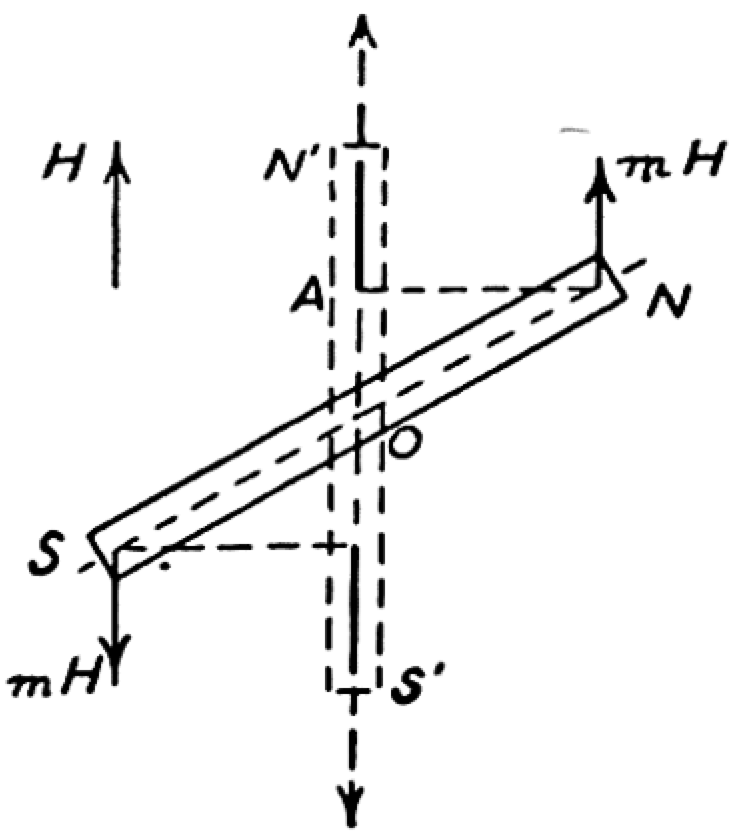
$\therefore$
$$\cos\theta=0{\cdot}486$$
$$\underline{\theta=60°\ 54'}$$

In the "end-on" position
$$F=\frac{2Md}{(d^2-l^2)^2} \text{ and } F=\mathbf{H}\tan\theta$$

$\therefore$
$$M=\frac{(d^2-l^2)^2\,\mathbf{H}\tan\theta}{2d}$$
$$=\frac{360000\times0{\cdot}18\times1{\cdot}7966}{50}$$
$$=\underline{2328 \text{ c.g.s. units}}$$

161.

How much work woud be done in turning a magnet of moment 500 c.g.s. units through an angle of 60° from its equilibrium position in the meridian if the value of H is 0·18 oersted?



Let the pole strength of the magnet be m and the distance between its poles $2l$.

Then the force acting on each pole due to the earth's field is mH dynes.

Referring to the diagram, the points of application of these forces have been moved from N′ to N and from S′ to S respectively in turning the magnet through 60°.

The component of NN′ parallel to the line of action of the force mH is N′A.

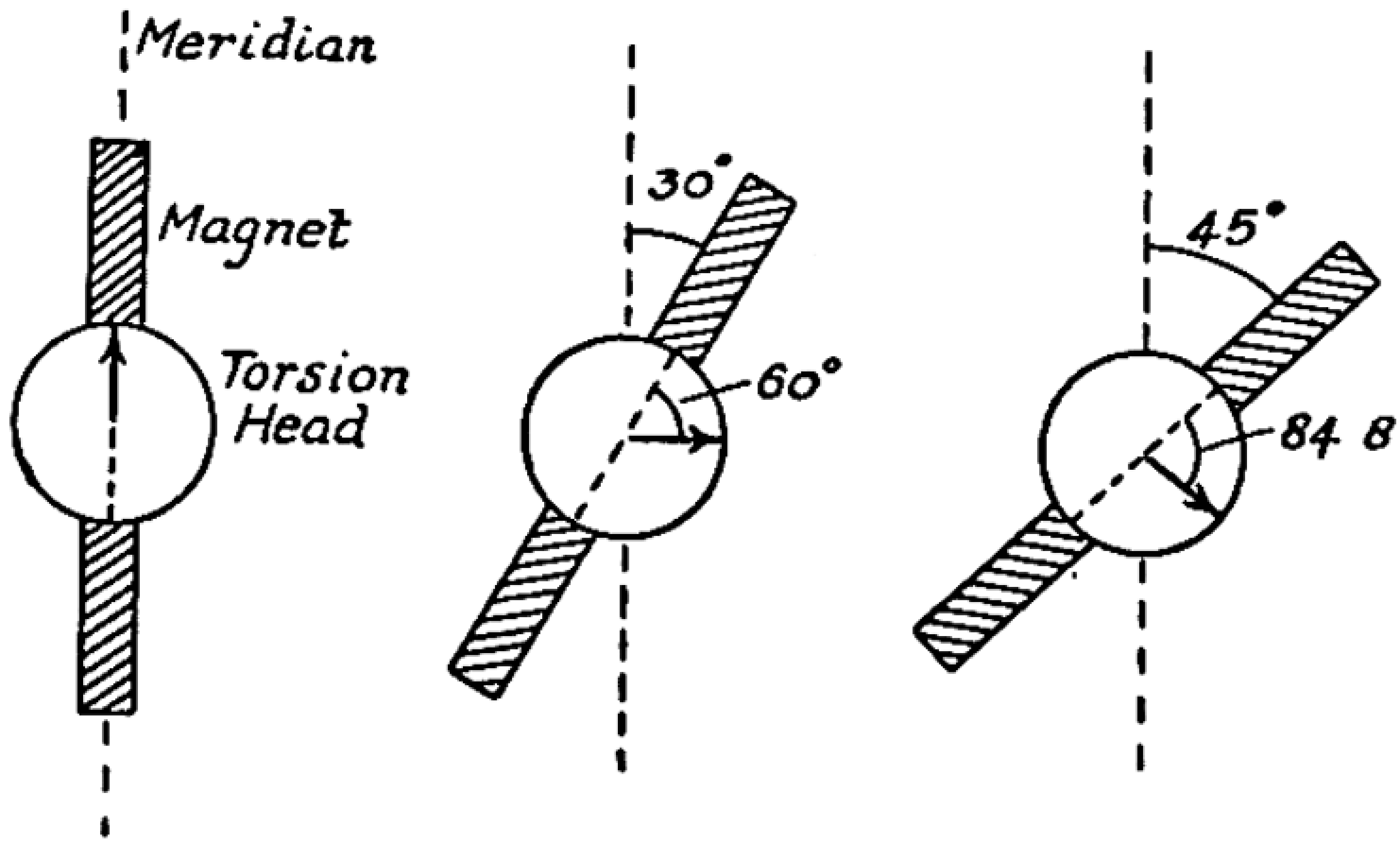
Therefore the work done on the north pole is
$$m\mathbf{H}\times \mathrm{N'A} \text{ ergs}$$
$$=m\mathbf{H}(\mathrm{N'O}-\mathrm{AO})$$
$$=m\mathbf{H}(l-l\cos 60°)$$
$$=ml\mathbf{H}(1-\cos 60°)$$

An equal amount of work is done on the south pole, and therefore the total work done in turning the magnet through 60°

$$=2mlH(1-\cos 60^\circ)$$
$$=MH(1-\cos 60^\circ)$$
$$=500\times 0{\cdot}18(1-0{\cdot}5)$$
$$=\underline{45 \text{ ergs}}$$

162.

A bar magnet is suspended horizontally with its axis in the meridian by a wire hanging from a graduated torsion head. In order to turn the magnet through 30°, the torsion head has to be turned through 90°. How much further must the head be turned in order that the axis of the magnet may lie at 45° to the meridian? (Assume that the torsional couple is proportional to the angle of twist.)



Angle of twist of wire in 30° position

$$=90^\circ-30^\circ$$
$$=60^\circ$$

If torsion head is turned through a further θ°, angle of twist of wire in 45° position

$$=90^\circ+\theta^\circ-45^\circ$$
$$=45^\circ+\theta^\circ$$

Couple required to hold a magnet of moment M at an angle a to the direction of a field of strength **H**

$$=M\mathbf{H}\sin a$$

$$\therefore \quad \frac{60}{45+\theta}=\frac{M\mathbf{H}\sin 30^\circ}{M\mathbf{H}\sin 45^\circ}$$

$$=\frac{0{\cdot}5}{1/\sqrt{2}}$$

$$=0{\cdot}707$$

$$60=0{\cdot}707(45+\theta)$$

$$\therefore \quad \theta=84{\cdot}8^\circ-45^\circ$$

$$=\underline{39{\cdot}8^\circ}$$

163.

What is the intensity of magnetisation in a bar magnet whose dimensions are $10\times1{\cdot}5\times0{\cdot}5$ cm. and whose magnetic moment is 800 dyne cm. per unit field?

Intensity of magnetisation = magnetic moment per unit volume

$$=\frac{800}{10\times1{\cdot}5\times0{\cdot}5}$$

$$=\frac{800}{7{\cdot}5}$$

$$=\underline{106{\cdot}7 \text{ c.g.s. units}}$$

164.

A vertical steel chimney is 10 metres high, 1 metre in diameter and its walls are 0·5 cm. thick. Calculate its magnetic moment due to the earth's vertical component from the following data:

$$\mathbf{H}=0{\cdot}18 \text{ oersted}$$

$$\text{Angle of dip}=60^\circ$$

$$\text{Susceptibility of steel}=12{\cdot}0$$

$$\text{Since } \tan\delta=\frac{\mathbf{V}}{\mathbf{H}}$$

$$\tan 60^\circ=\frac{\mathbf{V}}{0{\cdot}18}$$

$$\therefore \quad \mathbf{V}=0{\cdot}18\times\sqrt{3}$$

$$=0{\cdot}312 \text{ oersted}$$

This is the magnetising field, and therefore the susceptibility,

$$k=\frac{\mathbf{I}}{\mathrm{V}}$$

where $\mathbf{I}$ is the intensity of magnetisation.

$$\therefore \qquad 12=\frac{\mathbf{I}}{0{\cdot}312}$$

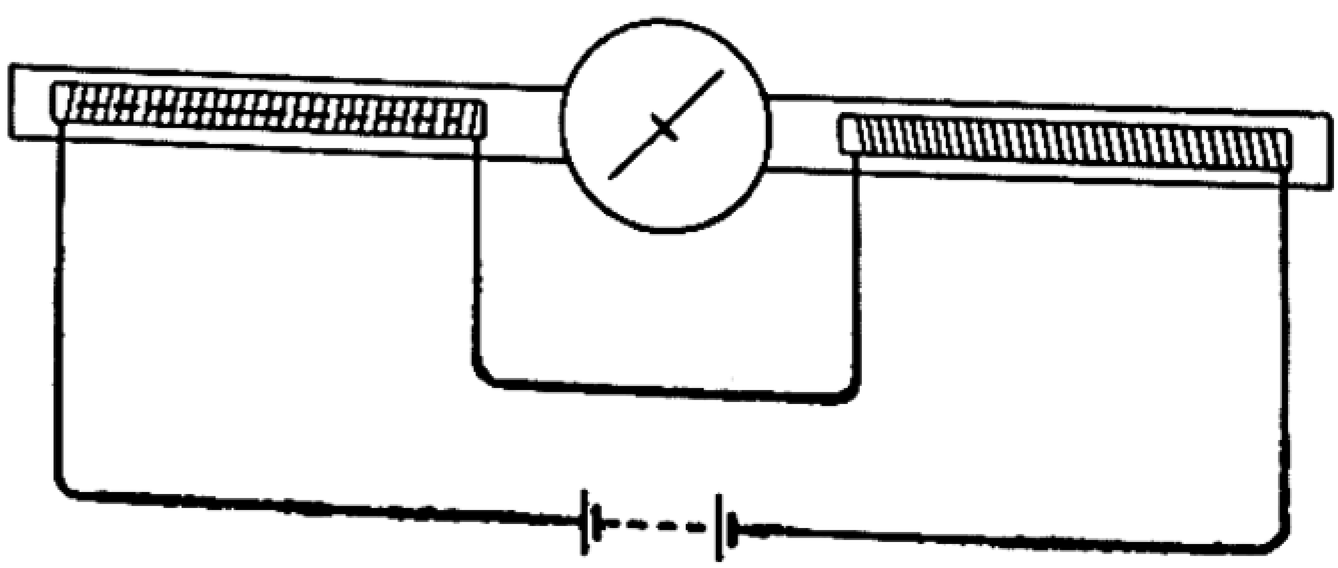
$$\mathbf{I}=3{\cdot}744 \text{ c.g.s. units}$$

$\mathbf{I}$ is the magnetic moment per unit volume, and therefore the moment of the chimney

$$=3{\cdot}744\times\text{the volume of steel in the chimney}$$
$$=3{\cdot}744\times1000\times100\pi\times0{\cdot}5$$
$$=\underline{588{,}000 \text{ dyne cm. per unit field}}$$

165.

An iron wire 24 cm. long and 2 sq. mm. in cross-section is placed symmetrically in a solenoid 30 cm. long containing 300 turns of wire. The solenoid is laid with its axis east and west along one arm of a deflection magnetometer and a similar solenoid joined in series with the first is placed along the other arm, the connections being such that when current flows, the field due to one solenoid at the compass needle is exactly neutralised by that due to the other. When the centre of the iron wire is 20 cm. from the centre of the needle and a current of 2 amp. flows, the needle is deflected through 45°. Calculate the intensity of magnetisation and the magnetic induction in the iron and also values of the permeability and susceptibility. Assume that the magnetic length of the wire is three-quarters of its actual length. H=0·18 oersted.



The *intensity of magnetisation,*

$$\mathbf{I}=\text{magnetic moment per unit volume}$$

$$=\frac{M}{24\times0{\cdot}02}=\frac{M}{0{\cdot}48}$$

In the "end-on" position

$$\frac{2Md}{(d^2-l^2)^2}=\mathbf{H}\tan\theta$$

$$\left.\begin{aligned} d&=20\text{ cm.}\\ l&=\frac{1}{2}\times\frac{3}{4}\times24=9\text{ cm.}\\ \tan\theta&=\tan45^\circ=1\\ \mathbf{H}&=0{\cdot}18\text{ oersted}\end{aligned}\right\} \quad \begin{aligned} M&=\frac{0{\cdot}18\times1\times(400-81)^2}{2\times20}\\ &=458{\cdot}1\text{ dyne cm. per unit field}\end{aligned}$$

$$\mathbf{I}=\frac{458{\cdot}1}{0{\cdot}48}$$

$$=\underline{954\text{ c.g.s. units}}$$

The *magnetic induction*

$$\mathbf{B}=\mathbf{H}+4\pi\mathbf{I}$$

$\mathbf{H}$, the magnetising field in the solenoid, is given by

$$\mathbf{H}=4\pi ni$$

where n is the number of turns per cm. and i is the current in absolute units.

$$\therefore \qquad \mathbf{H}=4\pi\times\frac{300}{30}\times\frac{2}{10}$$

$$=8\pi\text{ oersted}$$

$$\therefore \qquad \mathbf{B}=8\pi+(4\times954)\pi$$

$$=3824\pi$$

$$=\underline{12{,}010\text{ c.g.s. units (gauss)}}$$

Permeability,

$$\mu=\frac{\mathbf{B}}{\mathbf{H}}$$

$$=\frac{3824\pi}{8\pi}$$

$$=\underline{478}$$

Susceptibility,

$$k=\frac{\mathsf{I}}{\mathsf{H}}$$

$$=\frac{954}{8\times 3{\cdot}142}$$

$$=\underline{37{\cdot}9}$$

166.

What force is necessary to separate two similar bar magnets each of moment 900 c.g.s. units, length 9 cm. and cross-section 1 sq. cm., lying end to end with opposite poles in contact? Assume that the poles coincide with the ends of the magnet.

The pole strength m is given by

$$m=\frac{M}{2l}=\frac{900}{9}$$

$\therefore$ $m=100$ c.g.s. units

By Gauss's theorem the field strength close to a pole of strength m is $2\pi\frac{m}{a}$ oersted, where a is the area of the pole face.

The force experienced by the other pole of strength m in this field is

$$2\pi\frac{m}{a}\times m=2\pi\frac{m^2}{a} \text{ dynes}$$

$$=2\times 3{\cdot}142\times 100^2$$

$$=62{,}840 \text{ dynes}$$

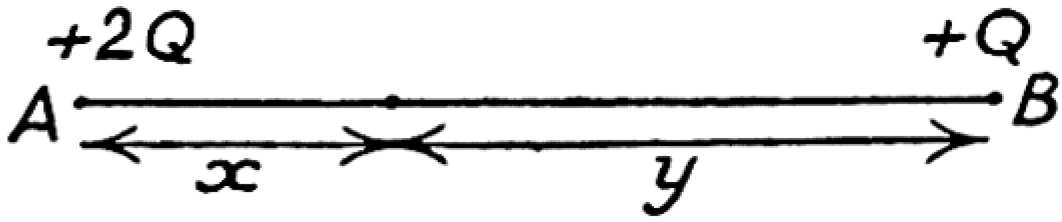
$$=\underline{64{\cdot}06 \text{ gm. wt.}}$$

and this force must just be exceeded to separate the magnets.

167.

Two point charges, one twice the other, are situated 15 cm. apart. Find the position of the point at which a unit point charge would remain in equilibrium (*a*) if the charges are of the same sign, (*b*) if they are of opposite sign.

(*a*)



If the unit point charge is in equilibrium, the forces on it due to the other two charges must be equal and opposite. This can only occur along the straight line joining the two charges—AB in the diagram.

Let the required point be distant x from A and y from B.

Then, applying the inverse square law:

$$\frac{2Q}{x^2}=\frac{Q}{y^2}$$

$$\therefore \qquad \frac{x}{y}=\sqrt{2} \text{ or } x=\sqrt{2}y$$

$$\text{Also} \qquad x+y=15$$

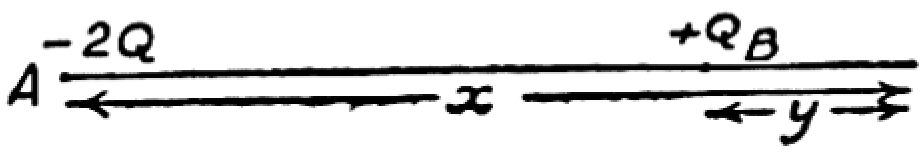
$$\therefore \qquad \sqrt{2}y+y=15$$

$$2{\cdot}414y=15$$

$$y=6{\cdot}21 \text{ cm.}$$

The point of equilibrium is between the charges on the straight line joining them, at a distance of 6·21 cm. from the smaller charge.

(*b*)



The neutral point must still be on the line joining the two charges, but it cannot now be between them if the forces they exert are to be oppositely directed. The point cannot be beyond A, since then it would be nearer to the greater charge, and could not be in equilibrium. Let it therefore be beyond B at a distance x from A and y from B.

Then as before $\frac{2Q}{x^2}=\frac{Q}{y^2}$ and $x=\sqrt{2}y$

Also

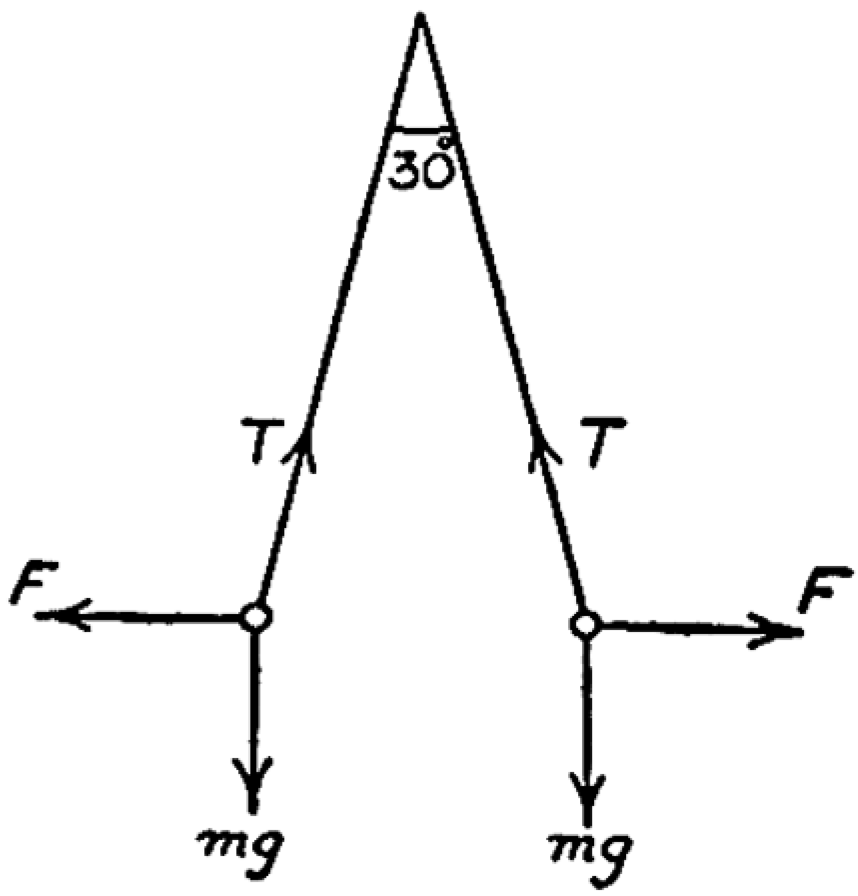
$$x-y=15$$
$$\sqrt{2}y-y=15$$
$$0{\cdot}414y=15$$
$$y=36{\cdot}2 \text{ cm.}$$

The point of equilibrium is on the straight line joining the charges at a distance of 36·2 cm. from the smaller charge, on the side opposite to the greater one.

(For an alternative method of working this type of problem, see Ex. 134.)

168.

Two pith balls each of mass 0·5 gm. are suspended from the same point by silk threads each 20 cm. long. Equal charges are given to the balls, which separate until the threads enclose an angle of 30°. Calculate the magnitude of the charge on each ball.



The three forces acting on each ball are its weight mg vertically down, the tension T in the thread acting in a direction of 15° to the vertical, and the horizontal force F due to repulsion between the charges. Calling the charges each Q and the distance between them d

$$F=\frac{Q^2}{d^2}$$

and since

$$d=2(20 \sin 15°)$$
$$=40\times 0{\cdot}2588$$
$$=10{\cdot}352 \text{ cm.}$$

$\therefore$ $$F=\frac{Q^2}{10{\cdot}35^2} \text{ dynes}$$

For equilibrium, the vertical component of T must equal mg, and its horizontal component must equal F.

$\therefore$ $$T \cos 15°=mg=0{\cdot}5\times 981$$

$$T \sin 15° = F = \frac{Q^2}{10 \cdot 35^2}$$

$$\therefore \qquad \tan 15° = \frac{Q^2}{0 \cdot 5 \times 981 \times 10 \cdot 35^2}$$

$$Q = \sqrt{0 \cdot 2679 \times 0 \cdot 5 \times 981 \times 10 \cdot 35^2}$$

$$= \underline{118 \cdot 7 \text{ c.g.s. units}}$$

169.

Two conducting spheres of radii 2 and 3 cm. carry charges of +18 and −6 c.g.s. units respectively. Find the charges which will remain on the spheres after they have been momentarily brought into contact.

In this type of problem, equations may be written down from the two rules:

(*a*) The algebraic sum of the charges remains the same before and after contact.

(*b*) The potentials of the conductors are equal after contact.

Let the final charges by Q_1 and Q_2 and the common potential V.

Then (*a*) gives $\quad Q_1 + Q_2 = 18 - 6 = 12$

$$\therefore \qquad Q_2 = 12 - Q_1$$

(*b*) gives

$$V = \frac{Q_1}{2} = \frac{Q_2}{3}$$

since the capacity of a sphere is numerically equal to its radius.

$$\therefore \qquad \frac{Q_1}{2} = \frac{12 - Q_1}{3}$$

$$3Q_1 = 24 - 2Q_1$$

$$\underline{Q_1 = 4 \cdot 8 \text{ c.g.s. units}}$$

$$\underline{Q_2 = 7 \cdot 2 \text{ c.g.s. units}}$$

or, from equation (*b*), we may write

$$Q_1 = \frac{2}{5}(Q_1 + Q_2) = 4 \cdot 8$$

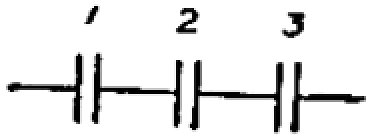
$$Q_2 = \frac{3}{5}(Q_1 + Q_2) = 7 \cdot 2$$

That is, <u>the total charge is shared in proportion to the capacities.</u>

170.

Three condensers have capacities of 1, 2 and 3 microfarads respectively. Calculate their combined capacities in all possible combinations employing all three condensers.

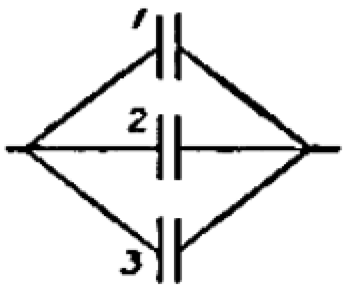
The following combinations are possible:

(1) 

In series: $\dfrac{1}{C_1}=\dfrac{1}{1}+\dfrac{1}{2}+\dfrac{1}{3}$

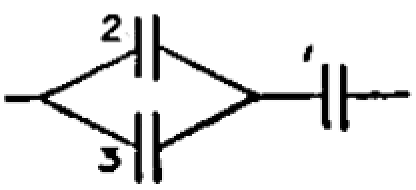
$=\dfrac{11}{6}$

$\underline{C_1=\tfrac{6}{11}\text{ mfd.}}$

(2) 

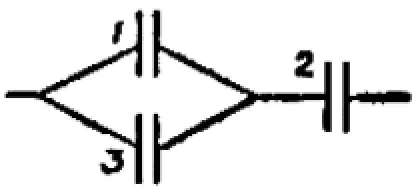
In parallel: $C_2=1+2+3$

$=\underline{6\text{ mfd.}}$

(3) 

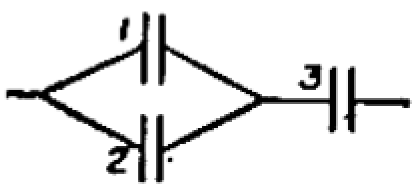
$\dfrac{1}{C_3}=\dfrac{1}{2+3}+\dfrac{1}{1}=\dfrac{6}{5}$

$\underline{C_3=\tfrac{5}{6}\text{ mfd.}}$

(4) 

$\dfrac{1}{C_4}=\dfrac{1}{1+3}+\dfrac{1}{2}=\dfrac{3}{4}$

$\underline{C_4=1\tfrac{1}{3}\text{ mfd.}}$

(5) 

$\dfrac{1}{C_5}=\dfrac{1}{1+2}+\dfrac{1}{3}=\dfrac{2}{3}$

$\underline{C_5=1\tfrac{1}{2}\text{ mfd.}}$

(6) 

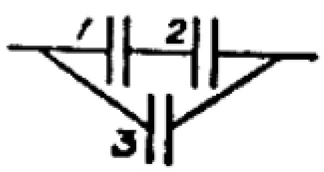
$$C_6=\cfrac{1}{\cfrac{1}{2}+\cfrac{1}{3}}+1$$

$=\underline{2\tfrac{1}{5}\text{ mfd.}}$

(7) 

$$C_7=\cfrac{1}{\cfrac{1}{1}+\cfrac{1}{3}}+2$$

$=\underline{2\tfrac{3}{4}\text{ mfd.}}$

(8) 

$$C_8=\cfrac{1}{\cfrac{1}{1}+\cfrac{1}{2}}+3$$

$=\underline{3\tfrac{2}{3}\text{ mfd.}}$

171.

What is the capacity of a condenser consisting of two metal discs 10 cm. in diameter and 1 mm. apart, the space between them being occupied by a dielectric of specific inductive capacity (dielectric constant) 5?

The capacity of a parallel-plate condenser is:

$$C=\frac{Ak}{4\pi d}$$

We are given $\begin{cases} A=\pi\times 5^2 \\ k=5 \\ d=0{\cdot}1 \text{ cm.} \end{cases}$

$$\therefore \qquad C=\frac{\pi\times 25\times 5}{4\times\pi\times 0{\cdot}1}$$

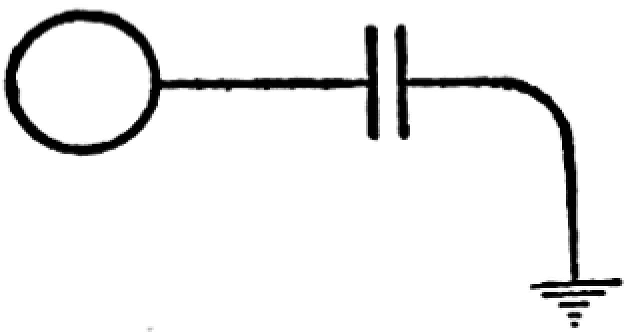
$$=\underline{312{\cdot}5 \text{ e.s.u.}}$$

172.

A parallel-plate air condenser of area 100 sq. cm. and plate separation 1 mm. has one plate earthed and the other connected to a conducting sphere of radius 1 cm. What charge must be given to the sphere to raise its potential to 100 c.g.s. units?

Capacity of condenser

$$=\frac{Ak}{4\pi d}$$



$$=\frac{100}{4\pi\times 0{\cdot}1}$$

$$=79{\cdot}55 \text{ c.g.s. units}$$

Capacity of sphere

$$=1 \text{ c.g.s. unit.}$$

Although there is an appearance of series connections in the arrangement, the total capacity is the sum of the individual capacities, since the charge is obviously shared between the sphere and the condenser, one plate of which must be at the same potential as the sphere.

Total capacity $=79{\cdot}55+1=80{\cdot}55$

Since $C=\frac{Q}{V}$, $\qquad Q=80{\cdot}55\times 100$

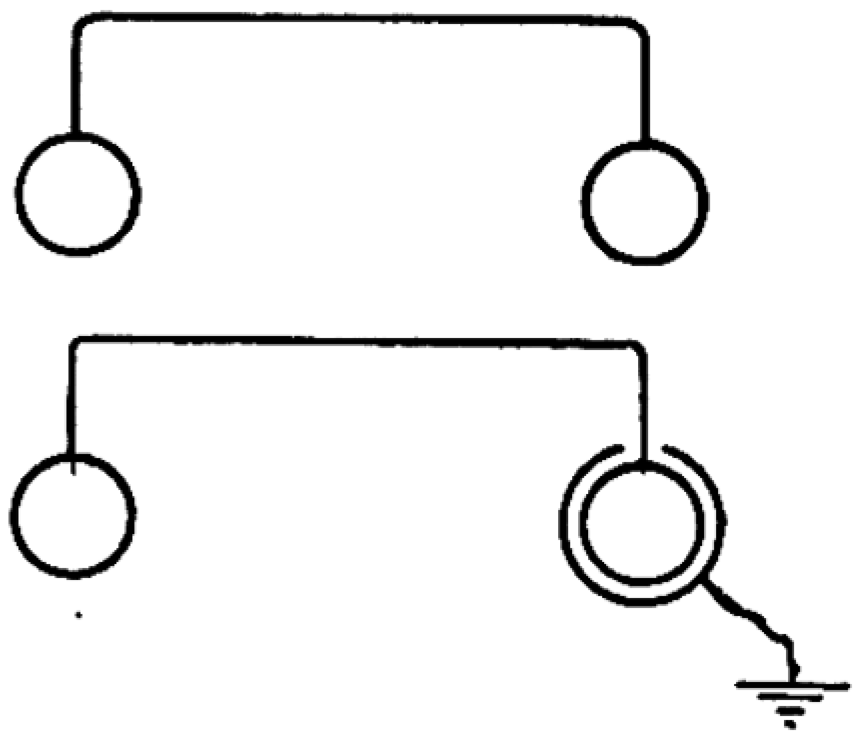
$$=\underline{8055 \text{ c.g.s. units}}$$

173.

Two insulated spherical conductors, each of radius 4 cm., are connected by a wire and charged to a potential of 100 volts. A spherical conducting shell of radius 5 cm., divided into hemispheres, is now fitted concentrically round one of the spheres and earthed, thus forming a spherical condenser, the wire joining the spheres passing through a small hole in the shell. Calculate the final potentials and charges on the two conductors, and the change produced in the electrical energy of the system. (1 e.s.u. of potential=300 volts.)

Initially, the charge on each sphere of capacity 4 is

$$\left(4\times\frac{100}{300}\right) \text{ e.s.u.}$$



giving a total charge of $\frac{8}{3}$ e.s.u.

The total final charge will have the same value and will be distributed in the same ratio as the capacities. (This follows from the condition that the final potentials are equal:

$$V=\frac{Q_1}{C_1}=\frac{Q_2}{C_2})$$

The capacity of a spherical condenser

$$=\frac{Rr}{R-r}$$
$$=\frac{4\times 5}{5-4}$$
$$=20$$

for the one under consideration.

We have therefore a conductor of capacity 4 connected to a condenser of capacity 20.

The total charge will be distributed in the ratio of

$$4 : 20 \text{ or } 1 : 5 \text{ or } \frac{1}{6} : \frac{5}{6}$$

so that

$$\left(\frac{1}{6}\times\frac{8}{3}\right)=\underline{\frac{4}{9}\text{ e.s.u. is the final charge on the sphere.}}$$

and

$$\left(\frac{5}{6}\times\frac{8}{3}\right)=\underline{2\frac{2}{9}\text{ e.s.u. is the final charge on the condenser.}}$$

$\underline{\text{The final common potential}}$ $=\frac{4}{9}\div4\left(\text{or } 2\frac{2}{9}\div20\right)$

$=\underline{\frac{1}{9}\text{ e.s.u.}}=\underline{33\frac{1}{3}\text{ volts}}$

Since the energy of charge $=\frac{1}{2}QV$

Initial energy $=\frac{1}{2}\left(\frac{8}{3}\times\frac{1}{3}\right)=\frac{4}{9}$ ergs

Final energy $=\frac{1}{2}\left(\frac{8}{3}\times\frac{1}{9}\right)=\frac{4}{27}$ ergs

Therefore the energy of the system has decreased by $\frac{8}{27}$ ergs.

174.

A condenser of capacity 0·2 microfarad is charged to a potential of 100 volts. A second condenser of much smaller capacity is now charged by connecting it momentarily across the terminals of the 0·2-mfd. condenser, the smaller condenser being then discharged by "short-circuiting" its terminals. This process of alternate charging and discharging is repeatedly performed, twenty times in all, when it is found that the potential of the larger condenser has fallen to 35 volts. What is the capacity of the smaller condenser?

If the required capacity is c, then the fraction of the charge on the 0·2-mfd. condenser removed by each operation is

$$\frac{c}{0{\cdot}2+c}$$

and the fraction remaining is

$$\frac{0{\cdot}2}{0{\cdot}2+c}$$

So after 20 sharings, the fraction remaining is

$$\left(\frac{0{\cdot}2}{0{\cdot}2+c}\right)^{20}=0{\cdot}35$$

$$\therefore \qquad \log\left(\frac{0{\cdot}2}{0{\cdot}2+c}\right)=\frac{1}{20}\log 0{\cdot}35$$

$$=\frac{\bar{1}{\cdot}5441}{20}=\frac{-0{\cdot}4559}{20}=-0{\cdot}02279$$

$$\therefore \qquad \log\left(\frac{0{\cdot}2+c}{0{\cdot}2}\right)=+0{\cdot}02279=\log 1{\cdot}054$$

so
$$\frac{0{\cdot}2+c}{0{\cdot}2}=1{\cdot}054$$

$$\underline{c=0{\cdot}0108 \text{ mfd.}}$$

175.

What value of magnetic field strength will be produced at the centre of a circular coil of wire of radius 5 cm. containing 20 turns, when a current of 20 milliamperes passes through it?

The magnetic intensity at the centre of a coil of n turns of radius r carrying a current of i c.g.s. units

$$=\frac{2\pi ni}{r}$$

$$n=20$$
$$r=5 \text{ cm.}$$
$$i=0{\cdot}020 \text{ amp.}=0{\cdot}0020 \text{ c.g.s. units}$$

$$\therefore \qquad \text{Magnetic intensity}=\frac{2\times 3{\cdot}142\times 20\times 0{\cdot}0020}{5}$$

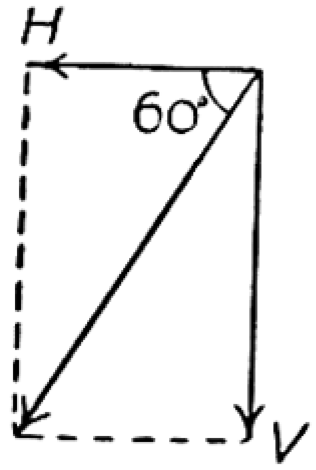
$$=\underline{0{\cdot}0503 \text{ oersted}}$$

176.

A dip needle, swinging in the magnetic meridian, is situated at the centre of a circular coil of wire having 10 turns of radius 20 cm., the angle of dip being 60° and the value of H 0·18 oersted. What current must be sent through the coil in order that the dip needle may come to rest with its axis horizontal?

If the needle sets horizontally, there can be no vertical component of the magnetic field. Therefore the field due to the

current in the coil must be equal and opposite to V, the earth's vertical component.



$$\mathbf{V}=\frac{2\pi ni}{r}$$

$$\mathbf{H}\tan 60°=\frac{2\times 3{\cdot}142\times 10\times i}{20}$$

$$i=\frac{20\times 0{\cdot}18\times\sqrt{3}}{2\times 3{\cdot}142\times 10}\text{ c.g.s. units}$$

$$=\frac{0{\cdot}18\times 1{\cdot}732}{3{\cdot}142}=0{\cdot}099\text{ c.g.s. units}$$

$$=\underline{0{\cdot}99\text{ amp.}}$$

177.

At a place where H=0·20 oersted a potential difference of 2 volts applied to the terminals of a tangent galvanometer produces a deflection of 65°. If the coil of the galvanometer has 500 turns of radius 7 cm., what is its resistance?

Let the resistance of the coil be R ohms.

Then we can write two expressions for the current I:

$$I=\frac{V}{R}=\frac{2}{R}$$

and

$$I=\frac{5r\mathbf{H}\tan\theta}{\pi n}=\frac{5\times 7\times 0{\cdot}2\times\tan 65°}{3{\cdot}142\times 500}$$

Equating these, $R=\dfrac{3142}{7\times 2{\cdot}145}$

$$=\underline{209\text{ ohms}}$$

178.

When a current of 0·1 amp. is sent through the coil of a tangent galvanometer, the needle, before settling down to a steady deflection of 45°, makes 10 oscillations in 12 sec. What will be the deflection and the period of oscillation if the current is reduced to 0·05 amp.?

Assuming that the plane of the coil is in the meridian,

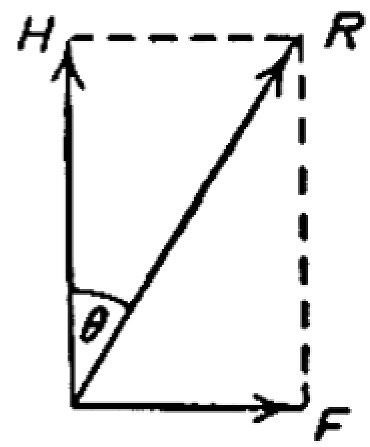
$$\frac{\tan\theta_1}{\tan\theta_2}=\frac{I_1}{I_2}$$

$$\tan\theta_2=\frac{0{\cdot}05}{0{\cdot}1}=0{\cdot}5$$

$$\therefore \qquad \underline{\theta_2=26°\,34'}$$

The field in which the needle swings is the resultant of **H**, the earth's field, and F, the field due to the current in the coil.

Since $$R=\frac{\mathbf{H}}{\cos\theta}$$



$$\frac{R_1}{R_2}=\frac{\cos\theta_2}{\cos\theta_1}=\frac{T_2{}^2}{T_1{}^2}$$

$$\frac{T_2{}^2}{1{\cdot}2^2}=\frac{\cos 26°\,34'}{\cos 45°}$$

$$\therefore \qquad T_2=1{\cdot}2\times\sqrt{\frac{0{\cdot}8945}{0{\cdot}7071}}$$

$$=\underline{1{\cdot}350\text{ sec.}}$$

179.

A constant current is sent through two tangent galvanometers, A and B, connected in parallel, the instruments having coils made of the same wire, but of different radii and different numbers of turns. If the deflections are 45° in A and 60° in B, find the ratio of the radii of the coils.

Let I_A be the current, R_A the resistance, n_A the number of turns, r_A the radius and θ_A the deflection for galvanometer A; and I_B, etc., for B.

Then, since for any two resistances in parallel, the ratio of the currents is the inverse of the ratio of the resistances (i.e. of the lengths of wire in the coils),

$$\frac{I_A}{I_B}=\frac{R_B}{R_A}=\frac{2\pi r_B n_B}{2\pi r_A n_A}$$

$$=\frac{r_B n_B}{r_A n_A}$$

Also, since for a tangent galvanometer

$$I=\frac{5r\mathrm{H}}{\pi n}\tan\theta$$

$$\frac{I_A}{I_B}=\frac{r_A n_B \tan\theta_A}{r_B n_A \tan\theta_B}$$

Equating the two expressions for the ratio of the currents,

$$\frac{r_B n_B}{r_A n_A}=\frac{r_A n_B \tan\theta_A}{r_B n_A \tan\theta_B}$$

$$\left(\frac{r_B}{r_A}\right)^2=\frac{\tan\theta_A}{\tan\theta_B}$$

$$=\frac{\tan 45°}{\tan 60°}$$

$$=\frac{1}{\sqrt{3}}=\frac{1}{1{\cdot}732}$$

$$\frac{r_B}{r_A}=\frac{1}{\sqrt{1{\cdot}732}}$$

$$=\frac{1}{1{\cdot}316}$$

Note that the answer is independent of the ratio of the numbers of turns.

180.

Two cells and a tangent galvanometer are connected in series and the galvanometer needle is deflected through 62° from the meridian. When the connections to one of the cells are reversed so that the cells are in opposition the deflection is 28°. What is the ratio of the E.M.F.s of the cells?

Let the E.M.F.s be E_1 and E_2, E_1 being the larger.

Let the current through the galvanometer be I_1 in the first case and I_2 in the second.

In both cases the total resistance in the circuit is the same, and therefore the currents are proportional to the E.M.F.s of the batteries.

When the cells are in series, battery E.M.F.$=E_1+E_2$

When the cells are in opposition, battery E.M.F.$=E_1-E_2$

$$\frac{E_1+E_2}{E_1-E_2}=\frac{I_1}{I_2}=\frac{\tan\theta_1}{\tan\theta_2}$$

$$\frac{E_1}{E_2}=\frac{\tan\theta_1+\tan\theta_2}{\tan\theta_1-\tan\theta_2}$$

$$=\frac{\tan 62°+\tan 28°}{\tan 62°-\tan 28°}$$

$$=\frac{1{\cdot}881+0{\cdot}532}{1{\cdot}881-0{\cdot}532}$$

$$=\frac{2{\cdot}413}{1{\cdot}349}$$

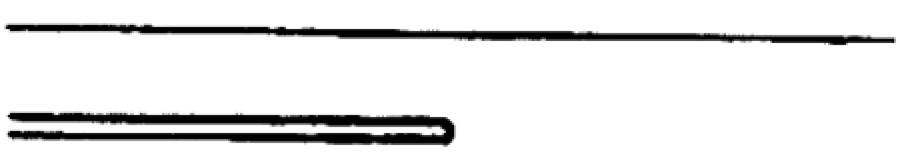
$$=\frac{1{\cdot}79}{1}$$

The ratio of the E.M.F.s of the cells is 1·79 : 1.

181.

A piece of wire of resistance 4 ohms is bent through 180° at its mid-point and the two halves twisted together. What is now its resistance?

When the wire is doubled, we have two wires, each of resistance 2 ohms, in parallel. Their combined resistance is given by



$$\frac{1}{R}=\frac{1}{2}+\frac{1}{2}=\frac{1}{1}$$

$$R=1 \text{ ohm}$$

From another point of view: the twisted wire is of one-half the length and double the cross-sectional area of the original wire. Therefore, since resistance is proportional to length and inversely proportional to cross-sectional area, the resistance is

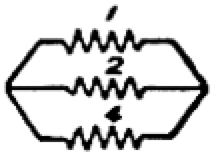
$$(4\div 2)\div 2=1 \text{ ohm}$$

182.

Find the effective resistances of all the possible combinations that can be obtained from three resistances of 1, 2 and 4 ohms respectively.

(1) 

$R_1 = 1+2+4 = \underline{7 \text{ ohms}}$

(2) 

$\frac{1}{R_2} = \frac{1}{1}+\frac{1}{2}+\frac{1}{4} = \frac{7}{4}$; $\underline{R_2 = \frac{4}{7} \text{ ohms}}$

(3) 

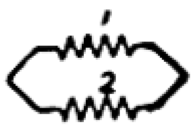
$R_3 = 1+2 = \underline{3 \text{ ohms}}$

(4) 

$R_4 = 1+4 = \underline{5 \text{ ohms}}$

(5) 

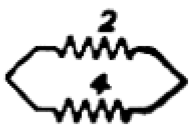
$R_5 = 2+4 = \underline{6 \text{ ohms}}$

(6) 

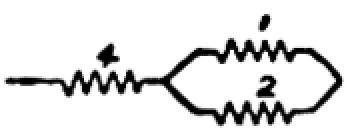
$\frac{1}{R_6} = \frac{1}{1}+\frac{1}{2} = \frac{3}{2}$; $\underline{R_6 = \frac{2}{3} \text{ ohms}}$

(7) 

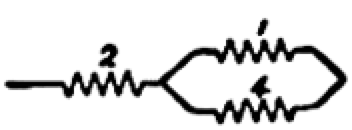
$\frac{1}{R_7} = \frac{1}{1}+\frac{1}{4} = \frac{5}{4}$; $\underline{R_7 = \frac{4}{5} \text{ ohms}}$

(8) 

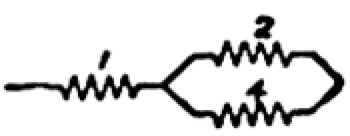
$\frac{1}{R_8} = \frac{1}{2}+\frac{1}{4} = \frac{3}{4}$; $\underline{R_8 = 1\frac{1}{3} \text{ ohms}}$

(9) 

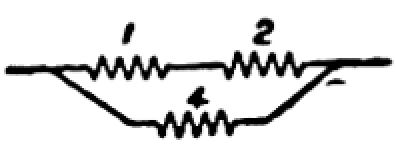
$R_9 = 4+R_6 = \underline{4\frac{2}{3} \text{ ohms}}$

(10) 

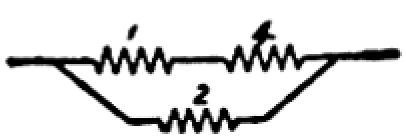
$R_{10} = 2+R_7 = \underline{2\frac{4}{5} \text{ ohms}}$

(11) 

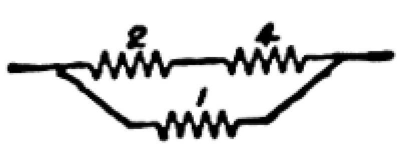
$R_{11} = 1+R_8 = \underline{2\frac{1}{3} \text{ ohms}}$

(12) 

$\frac{1}{R_{12}} = \frac{1}{3}+\frac{1}{4} = \frac{7}{12}$; $\underline{R_{12} = 1\frac{5}{7} \text{ ohms}}$

(13) 

$\frac{1}{R_{13}} = \frac{1}{5}+\frac{1}{2} = \frac{7}{10}$; $\underline{R_{13} = 1\frac{3}{7} \text{ ohms}}$

(14) 

$\frac{1}{R_{14}} = \frac{1}{6}+\frac{1}{1} = \frac{7}{6}$; $\underline{R_{14} = \frac{6}{7} \text{ ohms}}$

and, in addition, the three resistances taken separately.

183.

A resistance coil of nominal value 2 ohms is found to have an actual resistance of 2·12 ohms. What length of wire of resistance 20 ohms per metre must be connected in parallel with the coil in order that the combined resistance shall be exactly 2 ohms?

Let the required parallel resistance be R ohms.
Then from the law for resistances in parallel:

$$\frac{1}{2}=\frac{1}{R}+\frac{1}{2\cdot 12}$$

$$\frac{1}{R}=\frac{1}{2}-\frac{1}{2\cdot 12}$$

$$=\frac{2\cdot 12-2}{2\cdot 12\times 2}$$

$$=\frac{0\cdot 12}{4\cdot 24}$$

$$\therefore \qquad R=\frac{4\cdot 24}{0\cdot 12}$$

$$=35\cdot 33 \text{ ohms}$$

$$\text{Length of wire required}=\frac{35\cdot 33}{20}\text{ metres}$$

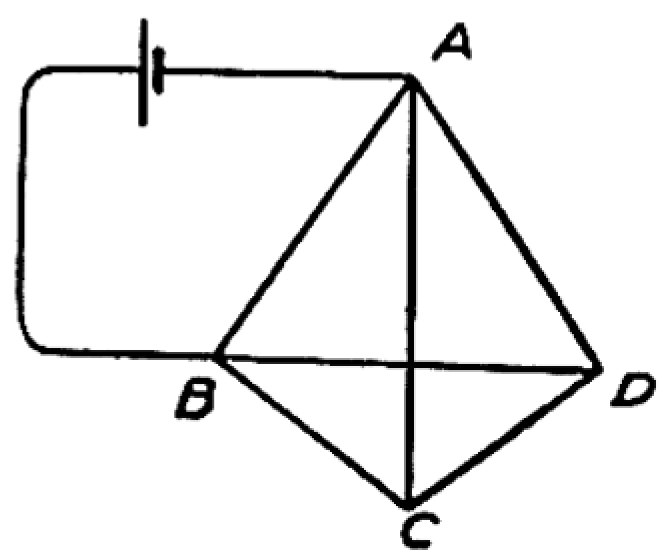
$$=\underline{176\cdot 7 \text{ cm.}}$$

Note that an error of 1 cm. in the length would cause an error of less than 0·001 ohm in the resistance of the coil.

184.

Six equal, straight wires, each of resistance 3 ohms, are joined so as to form the edges of a tetrahedron and two of the corners are connected to the terminals of a cell of E.M.F. 2 volts and negligible internal resistance. How much current flows through the cell?

Referring to the diagram, it is seen that the framework may be considered as three conductors in parallel: AB of resistance 3 ohms, ACB of resistance 6 ohms, and ADB of resistance 6 ohms, the points C and D being joined by a resistance of 3 ohms.



Now, the framework is symmetrical about a plane containing AB and bisecting CD at right angles; and by symmetry, the points C and D must be at the same potential, and therefore no current flows along CD. So far as this problem is concerned, then, CD may be ignored, and we are left with the three conductors of resistances 3, 6 and 6 ohms in parallel.

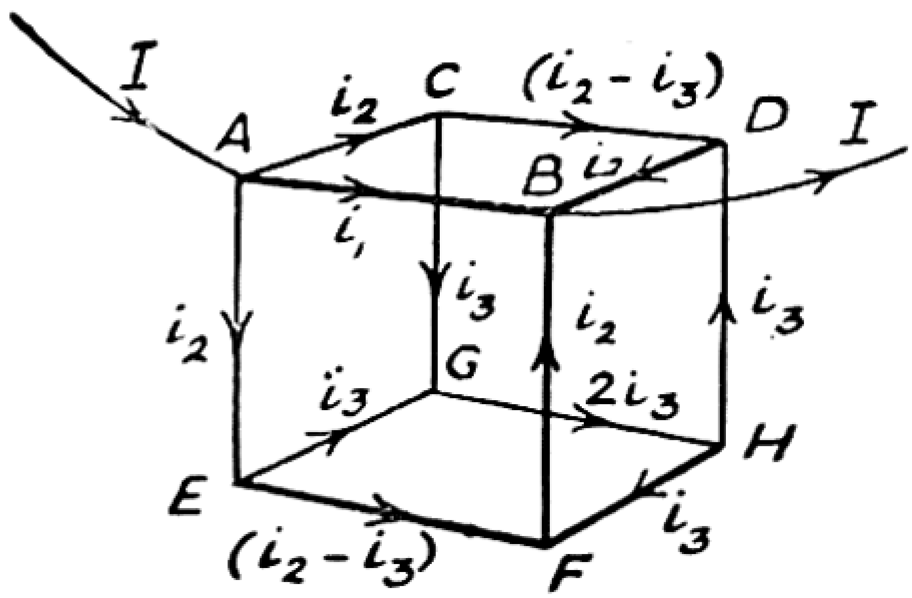
$$\text{Their combined resistance} = \frac{1}{\frac{1}{3}+\frac{1}{6}+\frac{1}{6}} = \frac{3}{2} \text{ ohms}$$

$$\text{and the total current} = \frac{2}{\frac{3}{2}}$$

$$= \underline{1\tfrac{1}{3} \text{ amp.}}$$

185.

A cubical framework is formed of twelve equal straight wires each of resistance 3 ohms, and a potential difference of 6 volts is applied between two adjacent corners. Calculate the total current flowing.



Let the current along AB be i_1.

From symmetry it is apparent that the currents along AC, AE, DB and FB will be equal. Call them each i_2.

Similarly, let i_3 represent the currents in CG, BG, HD and HF.

Then, applying Kirchoff's first law to the points G, C and E, the current along GH must be $2i_3$, and those along CD and EF each (i_2-i_3).

Applying the second law to the portions AB, ACDB and ACGHDB, we obtain:

(1) $6=3i_1$

(2) $6=3i_2+3(i_2-i_3)+3i_2=9i_2-3i_3$

$6=3i_2+3i_3+6i_3+3i_3+3i_2=6i_2+12i_3$

Elimination of i_3 from (2) and (3) gives

$$i_2=\frac{5}{7}\text{ amp.}$$

From (1) $\quad i_1=2$ amp.

The total current is the sum of the currents meeting at A or B—that is, i_1+2i_2.

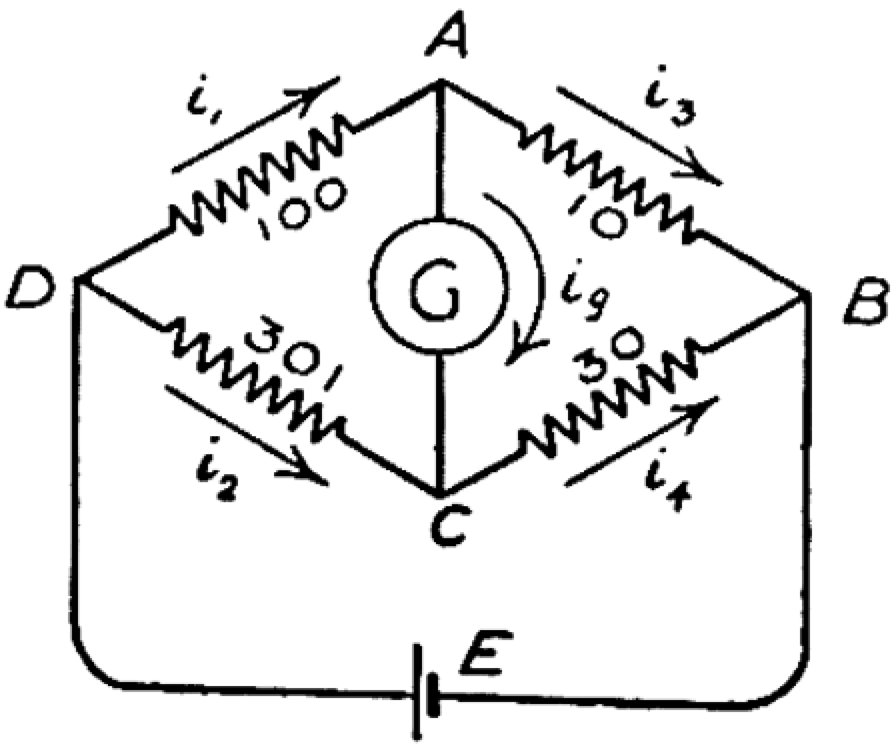
This equals $\qquad 2+\frac{10}{7}=\underline{3\tfrac{3}{7}\text{ amp.}}$

186.

For measuring an unknown resistance, a Post Office box is used in conjunction with a 2-volt accumulator and a galvanometer having a resistance of 20 ohms and a sensitivity of 1 division per micro-amp. With the ratio coils at 100 : 10, a balance is obtained when the box resistance is 300 ohms. What galvanometer deflection would result from taking a further 1-ohm plug out of the box?

The galvanometer current i_g is required. Let us suppose that it flows from A to C. From the Wheatstone bridge rule it is obvious that the unknown resistance must be equal to

$$300\times\frac{10}{100}=30\text{ ohms}$$



We can obtain a number of equations by applying Kirchoff's Laws to various portions of the circuit.

Applying the first law to the points A and C:

(1) $\qquad i_1=i_g+i_3$

(2) $\qquad i_4=i_g+i_2$

Applying the second law to the meshes ADC and ABC:

(3) $\qquad 301i_2=20i_g+100i_1$

(4) $\qquad 10i_3=30i_4+20i_g$ or $i_3=3i_4+2i_g$

Again applying the second law, this time to the meshes ABED and CBED, each of which contains the accumulator:

(5) $$2=100i_1+10i_3$$

(6) $$2=301i_2+30i_4$$

We now have six equations for determining 5 unknowns—a surplus of one equation—we shall not use (3).

Substituting in (5) and (6) values for i_1 and i_2 from (1) and (2):

(7) $$2=100i_g+110i_3$$

(8) $$2=331i_4-301i_g$$

We have now three equations (4), (7) and (8) containing 3 unknowns i_3, i_4 and i_g. By substituting for i_3 and i_4 in (4) values given by (7) and (8), we have:

$$\frac{(2-100i_g)}{110}=\frac{3(2+301i_g)}{331}+2i$$

$$\therefore \quad \frac{(2-100i_g)}{330}=\frac{(2+301i_g)}{331}+\frac{2}{3}i_g$$

$$\frac{2}{330}-\frac{2}{331}=i_g\left(\frac{301}{331}+\frac{2}{3}+\frac{100}{330}\right)$$

$$2\left(\frac{1}{330\times 331}\right)=i_g\left(\frac{301}{331}+\frac{320}{330}\right)$$

$$=i_g\left[\frac{(330\times 301)+(320\times 331)}{330\times 331}\right]$$

$$\therefore \quad i_g=\frac{2}{(330\times 301)+(320\times 331)}$$

$$=\frac{2}{330(301+321)}\text{ approx.}$$

$$=\frac{1}{330\times 311}$$

$$=\frac{1}{102630}\text{ amp.}$$

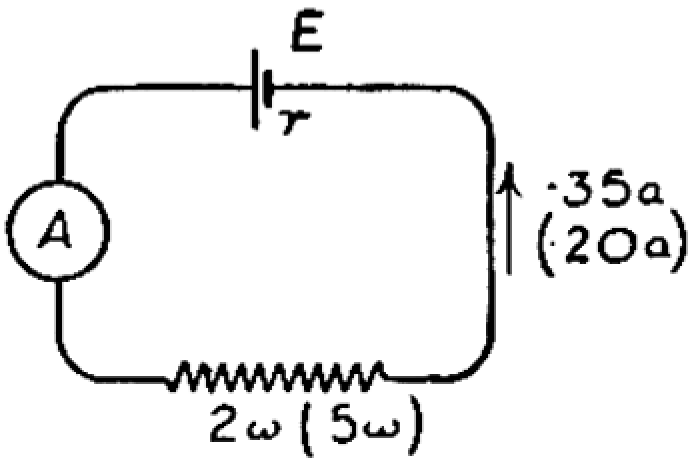
$$=\frac{10^6}{102630}\text{ micro-amp.}$$

$$=9{\cdot}71\text{ micro-amp.}$$

Therefore the galvanometer deflection $=9{\cdot}71$ divisions.

187.

When a cell has its terminals connected through an ammeter to a resistance of 2 ohms the ammeter indicates a current of 0·35 amp. Another resistance, of 3 ohms, is then joined in series with the original 2 ohms and the current is reduced to 0·20 amp. Calculate the E.M.F. and the internal resistance of the cell.



Let the E.M.F. be E volts and the internal resistance be r ohms. Then in the first case the total resistance in the circuit is $(2+r)$.

$\therefore \qquad E=0{\cdot}35(2+r)$

and in the second case $\qquad E=0{\cdot}20(2+3+r)$

$$\therefore \qquad 0{\cdot}35(2+r)=0{\cdot}20(5+r)$$
$$0{\cdot}7+0{\cdot}35r=1+0{\cdot}20r$$
$$r=\frac{0{\cdot}30}{0{\cdot}15}$$
$$=2$$

The internal resistance of the cell=2 ohms.

Substituting the value of r in the first equation gives

$$E=0{\cdot}35(2+2)$$
$$=1{\cdot}4$$

The E.M.F. of the cell=1·4 volts

NOTE.—In this and the two succeeding problems there are three variable quantities, the external resistance R, the current I and the terminal P.D. e. Both the E.M.F. of the cell and its internal resistance may be calculated if two values of each of two of these variables are given:

I_1, I_2 and R_1, R_2 as in this problem.

e_1, e_2 and R_1, R_2, as in Ex. 188.

e_1, e_2 and I_1, I_2, as in Ex. 189.

188.

The potential difference between the terminals of a cell is 1·2 volts when the cell is on open circuit and 1·0 volts when the terminals are joined by a resistance of 5 ohms. What is the internal resistance of the cell?

The first value of the P.D. tells us that the E.M.F. of the cell is 1·2 volts.

The second value, 1·0 volts, is the P.D. across the external resistance of 5 ohms, and therefore

$$1{\cdot}0=5\times I$$

(I being the current).

But the E.M.F. is the product of the current and the *total* resistance (internal+external) in the circuit.

$\therefore$
$$1{\cdot}2=(5+r)\times I$$

$\therefore$
$$\frac{1{\cdot}0}{1{\cdot}2}=\frac{5}{5+r}$$

whence
$$\underline{r=1\text{ ohm}}$$

The internal resistance of the cell=1 ohm.

189.

Find the E.M.F. and the internal resistance of a battery if the terminal P.D. is 28·5 volts when giving a current of 1 amp., and 27 volts when giving a current of 2 amp.

Let the E.M.F. be E volts and the internal resistance r ohms, the external resistances being R_1 and R_2 ohms respectively.

Then, since the terminal P.D. is the P.D. across the external resistance

$$28{\cdot}5=R_1\times 1$$
$$27{\cdot}0=R_2\times 2$$

The E.M.F. is the product of the current and the sum of the internal and external resistances

$\therefore$
$$E=(R_1+r)\times 1$$

and
$$E=(R_2+r)\times 2$$
$$(R_1+r)=2(R_2+r)$$

Substitution of the values of R_1 and R_2 derived above gives

$$(28{\cdot}5+r)=2(13{\cdot}5+r)$$

$\therefore$
$$r=1{\cdot}5\text{ ohms}$$

The internal resistance of the battery=1·5 ohms.

By inserting this value for r in one of the expressions for E, we obtain

$$E=(28{\cdot}5+1{\cdot}5)$$
$$E=30 \text{ volts}$$

The E.M.F. of the battery=30 volts.

190.

A length of copper wire of mass 4·5 kgm. has a resistance of 14·7 ohms. Calculate the length and diameter of the wire.

Density of copper=8·93 gm. per c.c.

Resistivity of copper=$1{\cdot}8\times 10^{-6}$ ohm. cm.

The two equations necessary to determine the two unknowns (length and diameter) are obtained by writing down expressions for the mass and resistance of the wire.

Let the length and diameter be L and d respectively. Then

$$\text{Mass}=8{\cdot}93\times\pi\times\frac{d^2}{4}\times L=4500 \text{ gm.}$$

$$\text{Resistance}=1{\cdot}8\times 10^{-6}\times L\times\frac{4}{\pi d^2}=14{\cdot}7 \text{ ohms}$$

$$\therefore \qquad d^2L=\frac{4\times 4500}{8{\cdot}93\times\pi}$$

$$\frac{L}{d^2}=\frac{14{\cdot}7\times\pi}{1{\cdot}8\times 10^{-6}\times 4}$$

From which $L=\sqrt{d^2L\times\frac{L}{d^2}}=\sqrt{\frac{4\times 4500\times 14{\cdot}7\times\pi}{8{\cdot}93\times\pi\times 1{\cdot}8\times 10^{-6}\times 4}}$

$$=\sqrt{41{\cdot}2\times 10^8}=6{\cdot}42\times 10^4 \text{ cm.}$$

$$L=642 \text{ metres}$$

Substituting this value in the first equation:

$$d=\sqrt{\frac{4\times 4500}{8{\cdot}93\times 3{\cdot}142\times 6{\cdot}42\times 10^4}}$$

$$=0{\cdot}10 \text{ cm.}$$

$$d=1 \text{ mm.}$$

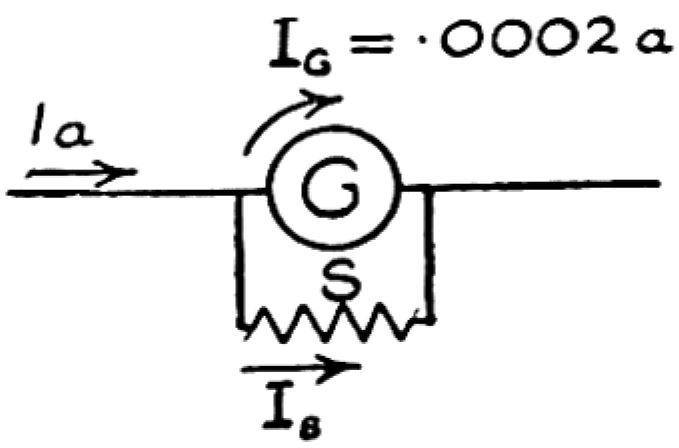
191.

A galvanometer having a resistance of 200 ohms gives a full-scale deflection for a current of 0·2 milliamp. Show, with the aid

of diagrams, how you would adapt the instrument for use as (*a*) an ammeter having a range of 0 to 1 amp., (*b*) a voltmeter to read from 0 to 100 volts.

In each case the current through the galvanometer must be equal to 0·2 milliamp. when the instrument is giving its maximum reading.

(*a*) In an *ammeter*, which must have a low resistance, the excess current is by-passed by means of a shunt—that is, a resistance connected in parallel with the galvanometer.



For the maximum reading of 1 amp., 0·0002 amp. will pass through the galvanometer and the remaining (1—0·0002) amp. through the shunt. Since G and S are in parallel

$$\therefore \quad \frac{S}{G}=\frac{I_G}{I_s}$$

$$\therefore \quad \frac{S}{200}=\frac{0{\cdot}0002}{0{\cdot}9998}$$

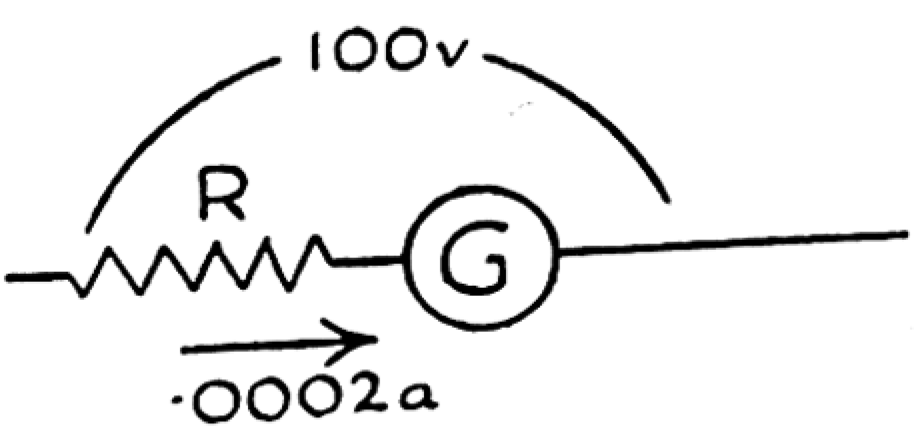
$$S=\frac{200}{4999}$$

$$=\underline{0{\cdot}040 \text{ ohms}}$$

For use as an ammeter having a range 0 to 1 amp., the galvanometer must be provided with <u>a shunt of resistance 0·040 ohms.</u>

(*b*) In a *voltmeter*, which should have a high resistance, the current is restricted by a series resistance.

Total resistance$=200+R$



$$0{\cdot}0002=\frac{100}{200+R}$$

$$R=\frac{100}{0{\cdot}0002}-200$$

$$=500{,}000-200$$

$$=\underline{499{,}800 \text{ ohms}}$$

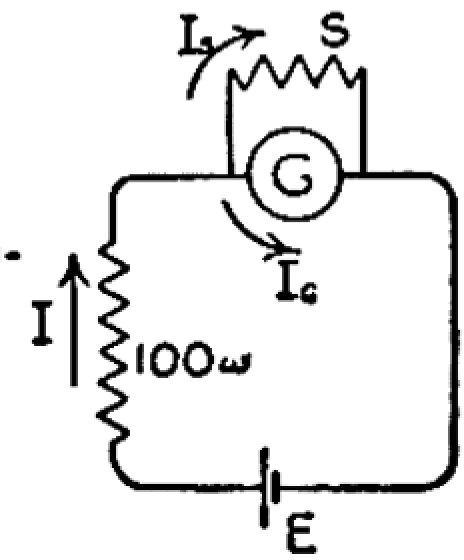
For use as a voltmeter having a range 0 to 100 volts, <u>a resistance of 499,800 ohms must be connected in series with the galvanometer.</u>

192.

A galvanometer provided with a variable shunt is connected in series with a resistance box and a cell of negligible internal resistance. When the series resistance is 100 ohms, it is found that with shunt resistances of 10 and 50 ohms, the deflections are respectively 9 and 30 divisions. What is the resistance of the galvanometer?

The current flowing through the cell is

$$I=\frac{E}{100+\frac{SG}{S+G}}$$



The current flowing through the galvanometer is

$$I_G=\frac{S}{S+G}\left(\frac{E}{100+\frac{SG}{S+G}}\right)$$

So, assuming that the galvanometer current is proportional to the deflection,

$$\frac{9}{30}=\frac{10}{10+G}\cdot\frac{50+G}{50}\left(\frac{100+\frac{50G}{50+G}}{100+\frac{10G}{10+G}}\right)$$

$$=\frac{100(50+G)+50G}{500(10+G)+50G}=\frac{5000+150G}{5000+550G}$$

$\therefore$ $15000+450G=4500+495G$

$\underline{G=233 \text{ ohms}}$

193.

Current passes through copper and water voltameters connected in series. What volume of hydrogen at a pressure of 77·5 cm. of mercury and at 15° C. will be collected in the time taken to deposit 1·5 gm. of copper?

Atomic weight of copper=63

Density of hydrogen at N.T.P.=0·09 gm./litre.

One of Faraday's laws of electrolysis states that the mass of an element liberated is proportional to its chemical equivalent.

The equivalent of copper is 63/2, since it is divalent.

Therefore, calling the mass of hydrogen m gm.,

$$\frac{1\cdot5}{m}=\frac{63}{2}$$

$$m=\frac{3}{63}\text{ gm.}$$

The volume of this at N.T.P. $=\dfrac{3}{63\times0\cdot09}$ litres

and at 77·5 cm. and 15° C. the volume will be

$$\left(\frac{3}{63\times0\cdot09}\times\frac{288}{273}\times\frac{760}{775}\right)\text{ litres}$$

$$=0\cdot547\text{ litres}$$

The volume of hydrogen collected will be 547 c.c.

194.

A tangent galvanometer having a coil consisting of two circular turns of wire of radius 7 cm. is connected in series with a copper voltameter and a battery. The current causes a deflection of 60° in the galvanometer and deposits 1·05 gm. of copper on the cathode of the voltameter in 30 minutes. What is the horizontal intensity of the earth's magnetic field?

E.C.E. of copper=0·000329 gm. per coulomb.

The current I may be calculated from the voltameter data:

$$I=\frac{1\cdot05}{0\cdot000329\times30\times60}$$

$$=1\cdot775\text{ amp.}$$

For the tangent galvanometer:

$$I=\frac{5r\mathbf{H}}{\pi n}\tan\theta$$

$$\therefore\qquad 1\cdot775=\frac{5\times7\times\mathbf{H}\times1\cdot732}{3\cdot142\times2}$$

and $\qquad\mathbf{H}=0\cdot184$ oersted

195.

How many watts are equivalent to 1 horse-power? (453·6 gm. =1 lb.; 2·54 cm.=1 in.)

1 watt=1 joule per sec.

$$
\begin{aligned}
1 \text{ horse-power} &= 550 \text{ ft. lb. per sec.}\\
&= (550 \times 12 \times 2{\cdot}54) \text{ cm. lb. per sec.}\\
&= (550 \times 12 \times 2{\cdot}54 \times 453{\cdot}6) \text{ cm. gm. per sec.}\\
&= (550 \times 12 \times 2{\cdot}54 \times 453{\cdot}6 \times 981) \text{ cm. dynes or ergs per sec.}\\
&= \left(\frac{550 \times 12 \times 2{\cdot}54 \times 453{\cdot}6 \times 981}{10^7}\right) \text{ joules per sec., or watts}\\
&= \underline{746 \text{ watts}}
\end{aligned}
$$

196.

How long will it take to heat 2 litres of water from 25° to 100° C. in an electric kettle taking 5 amp. from a 220-volt supply, and what would it cost at 1*d.* per kilowatt-hour? ($J=4{\cdot}18$ joules per cal.)

Heat required $=(2000 \times 75)$ cal.

This is equivalent to $(2000 \times 75 \times 4{\cdot}18)$ joules or watt-sec.

which equals $\left(\dfrac{2000 \times 75 \times 4{\cdot}18}{1000}\right)$ kilowatt-sec.

$$\left(\frac{2000 \times 75 \times 4{\cdot}18}{1000 \times 3600}\right) \text{ kw.-hours}$$

$$=0{\cdot}174 \text{ kw.-hour}$$

and <u>the cost is therefore 0·174*d.*</u>

The electrical energy used $=IVt$ joules

$$\therefore \qquad 2000 \times 75 \times 4{\cdot}18 = 5 \times 220 \times t$$

$$
\begin{aligned}
t &= \left(\frac{2000 \times 75 \times 4{\cdot}18}{5 \times 220}\right) \text{ sec.}\\
&= 570 \text{ sec.}\\
&= \underline{9\tfrac{1}{2} \text{ min.}}
\end{aligned}
$$

197.

By means of an electric immersion heater connected to 240-volt mains, it is required to heat 100 litres of water from 20° to 70° C. in 2 hours. Assuming no heat is lost, calculate the current necessary to effect this and the resistance of the heating coil. ($J=4{\cdot}18$ joules per cal.)

Heat required to raise the temperature of 100 litres of water from 20° C. to 70° C.

$$=(100\times1000\times50) \text{ cal.}$$

Electrical energy equivalent to this

$$=(4{\cdot}18\times5\times10^6) \text{ joules}$$

This has to be supplied in 2 hours.

$$\therefore \qquad \text{Power required}=\left(\frac{4{\cdot}18\times5\times10^6}{2\times3600}\right) \text{ joules per sec. or watts}$$

If I is the current,
$$240I=\frac{4{\cdot}18\times5\times10^6}{2\times3600}$$

$$\therefore \qquad I=\left(\frac{4{\cdot}18\times5\times10^6}{2\times3600\times240}\right) \text{ amp.}$$

$$=\frac{20900}{1728}$$

$$=\underline{12{\cdot}1 \text{ amp.}}$$

The resistance of the coil must be $\dfrac{240}{12{\cdot}1}$

$$=\underline{19{\cdot}8 \text{ ohms.}}$$

198.

How long will it take to raise the temperature of the air in a room $4\times5\times3$ metres through 10° C. by means of an electric heater taking 5 amp. from 250-volt mains, assuming no heat loss?

Density of air$=0{\cdot}00129$ gm./c.c.

Specific heat of air at constant pressure$=0{\cdot}24$.

Joule's equivalent$=4{\cdot}2$ joules per cal.

Mass of air $=0{\cdot}00129\times4\times5\times3\times10^6$ gm.

Heat required $=0{\cdot}00129\times60\times10^6\times10\times0{\cdot}24$ cal.

Electrical energy required$=0{\cdot}00129\times6\times10^8\times0{\cdot}24\times4{\cdot}2$ joules

Time taken
$$=\frac{0{\cdot}00129\times6\times10^8\times0{\cdot}24\times4{\cdot}2}{250\times5\times60} \text{ min.}$$

$$=\underline{10{\cdot}4 \text{ min.}}$$

199.

What resistance must be connected in series with a 100-watt, 110-volt lamp if it is to be used, taking its rated current, on a 240-volt circuit?

Since Power (watts) = Voltage × Current (amps.)

The rated current taken by the lamp $= \frac{100}{110}$ amp.

This current will be passed on a 240-volt circuit by a resistance of

$$\left(240 \div \frac{100}{110}\right) \text{ ohms.}$$
$$= 264 \text{ ohms.}$$

The resistance of the lamp is its rated voltage divided by its rated current:

$$\left(110 \div \frac{100}{110}\right) = 121 \text{ ohms.}$$

So the added resistance must be (264 − 121) ohms.

$$= \underline{143 \text{ ohms.}}$$

200.

How many 60-watt lamps may be safely run on a 230-volt circuit fitted with a 5-amp. fuse?

To avoid blowing the fuse, the total current must not exceed 5 amp. Each lamp takes a current of $\frac{60}{230}$ amp.

5 amperes would be taken by $\left(5 \div \frac{60}{230}\right)$ lamps.

That is, by 19·2 lamps

Therefore 19 lamps could safely be run on the 5-amp. circuit.

(In practice a bigger margin would be allowed.)

201.

A consumer wishes to use electric power equal to 4 kilowatts at 100 volts, the resistance of the mains connecting his plant to the power station being 0·5 ohms. What potential difference must

be applied between the mains at the power station and what percentage of the power generated is lost in heat in the mains?

The current in all parts of the circuit must be the same—namely, $\frac{4000}{100}$ amp.

The potential drop due to the passage of this current of 40 amp. through the mains of resistance 0·5 ohms is equal to

$$40\times0{\cdot}5=20 \text{ volts}$$

Therefore an additional 20 volts must be applied at the power station if the consumer is to receive 100 volts.

Potential difference between mains at power station=120 volts.

$$\begin{aligned}\text{Power lost in heating mains}&=I^2R\\&=40^2\times0{\cdot}5 \text{ watts}\\&=0{\cdot}80 \text{ kilowatts}\end{aligned}$$

This is $\left(\frac{0{\cdot}8}{4{\cdot}8}\times100\right)$ per cent. of the total power supplied by the station.

$16\frac{2}{3}$ per cent. of the power is lost in the mains.

202.

A 60-watt, 100-volt tungsten filament lamp has a resistance of 13·35 ohms at 20° C. What will its temperature be when connected to the 100-volt mains? The temperature coefficient of resistance for tungsten is 0·0054 per deg. C. (a mean value taken over the temperature range 0° to 2500° C.).

When the P.D. across the filament is 100 volts, the power consumed is 60 watts. Therefore the current through it is

$$I=\frac{60}{100}=0{\cdot}60 \text{ amp.}$$

and the resistance of the filament is

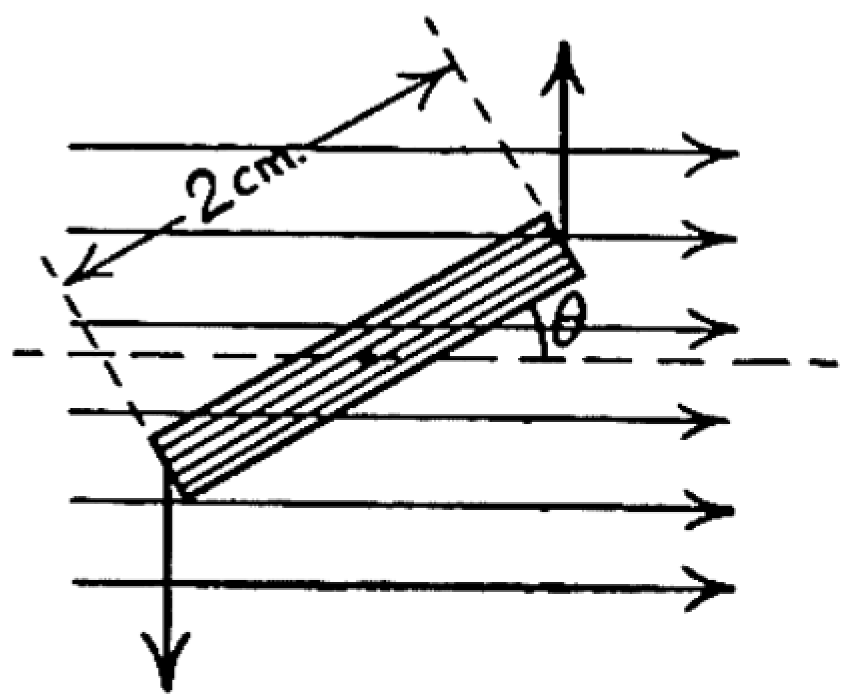
$$\frac{100}{0{\cdot}6}=166{\cdot}7 \text{ ohms}$$

at its working temperature, say t° C.

So we have $$R_t = 166{\cdot}7 = R_0(1 + {\cdot}0054t)$$
and $$R_{20} = 13{\cdot}35 = R_0(1 + {\cdot}0054 \times 20)$$
$$= R_0(1 + 0{\cdot}108)$$
$$\frac{R_t}{R_{20}} = \frac{166{\cdot}7}{13{\cdot}35} = \frac{1 + {\cdot}0054t}{1{\cdot}108}$$
$$166{\cdot}7 + (166{\cdot}7 \times 0{\cdot}108) = 13{\cdot}35 + 0{\cdot}0721t$$
$$166{\cdot}7 + 18{\cdot}0 - 13{\cdot}35 = 0{\cdot}0721t$$
$$t = \frac{171{\cdot}35}{{\cdot}0721}$$
$$= \underline{2377^\circ \text{ C.}}$$

203.

The coil of a moving-coil, mirror galvanometer consists of 100 turns of fine wire wound on a rectangular former 2 cm. wide by 3 cm. long, and is suspended in a uniform field of strength 600 oersted by a wire which requires a couple of 1 dyne cm. to twist the coil through 1 radian. What current through the galvanometer will produce a deflection of 50 cm. in the spot of light on a scale 1 metre away?



Let the required current be I amp., and the angular deflection of the coil θ radians.

Then the force acting on each side of the coil of length 3 cm. is

$$\left(3 \times 600 \times \frac{I}{10}\right) \text{ dynes per turn}$$

or $(3 \times 100 \times 60I)$ dynes for the 100 turns in a direction at right angles to the field.

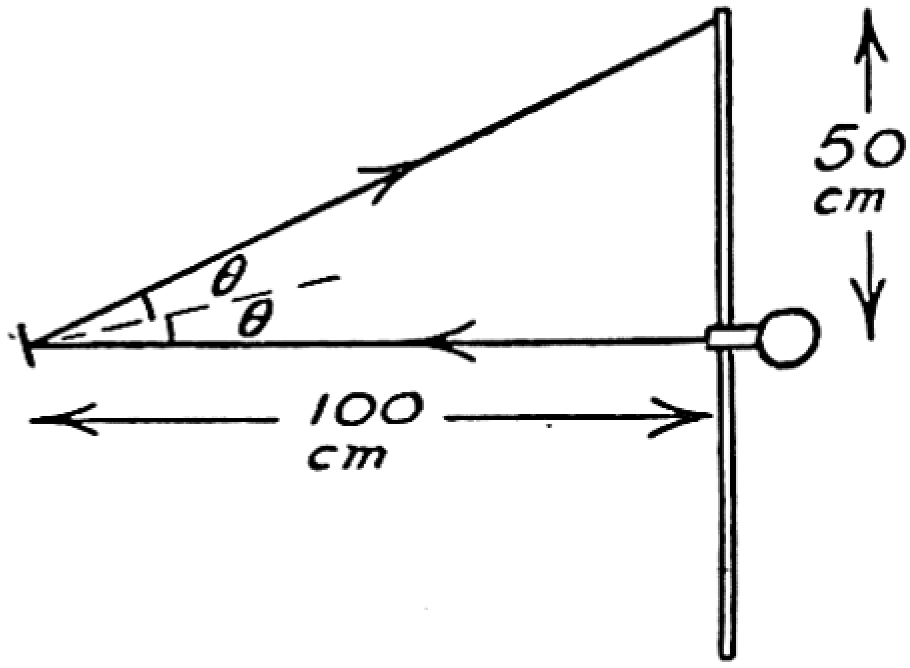
The two equal forces acting on the two sides of the coil constitute a couple of moment

$$(3 \times 100 \times 60I \times 2 \times \cos\theta) \text{ dyne cm.}$$

which is in equilibrium with the couple θ brought into action by twisting the suspending wire.

$$\therefore \qquad \theta = 36000 \cos\theta \times I$$

The angular deflection of the reflected light beam is 2θ and, for a deflection of 50 cm.,



$$\tan 2\theta = \frac{50}{100} = 0{\cdot}50$$

$$\therefore 2\theta = 26^\circ\ 30'$$

and $\quad \theta = 13^\circ\ 15'$

$$= \frac{13{\cdot}25 \times 2\pi}{360} \text{ radians}$$

$$= 0{\cdot}231 \text{ radian}$$

Substituting in the first equation

$$0{\cdot}231 = 36000 \times I \times \cos 13^\circ\ 15'$$

$$= 36000 \times I \times 0{\cdot}974$$

$$\therefore \qquad \underline{I = 6{\cdot}59 \times 10^{-6} \text{ amp.}}$$

204.

Along a portion of track running magnetic north and south a train travels at a speed of 80 km. per hour. Calculate the E.M.F. in volts generated between the ends of the axles which are 1·7 metres long, **H** being taken as 0·18 oersted and the angle of dip as 65°.

Induced E.M.F. = rate of cutting of lines of force

$$= \mathbf{V} \times (\text{area swept out by axle in one second})$$

$$= \mathbf{V} \times (\text{length of axle}) \times (\text{velocity})$$

$$= \frac{0{\cdot}18 \times \tan\ 65^\circ \times 170 \times 80 \times 10^5}{3600} \text{ e.m.u.}$$

$$= 1{\cdot}457 \times 10^5 \text{ e.m.u.}$$

$$= \underline{1{\cdot}457 \times 10^{-3} \text{ volts}}$$

205.

A rectangular coil of 20 turns, each of area 800 sq. cm., is rotated uniformly 5 times per second in a magnetic field of strength 15 oersted, the axis of rotation being at right angles to the direction of the field. Calculate the maximum E.M.F. induced in the coil.

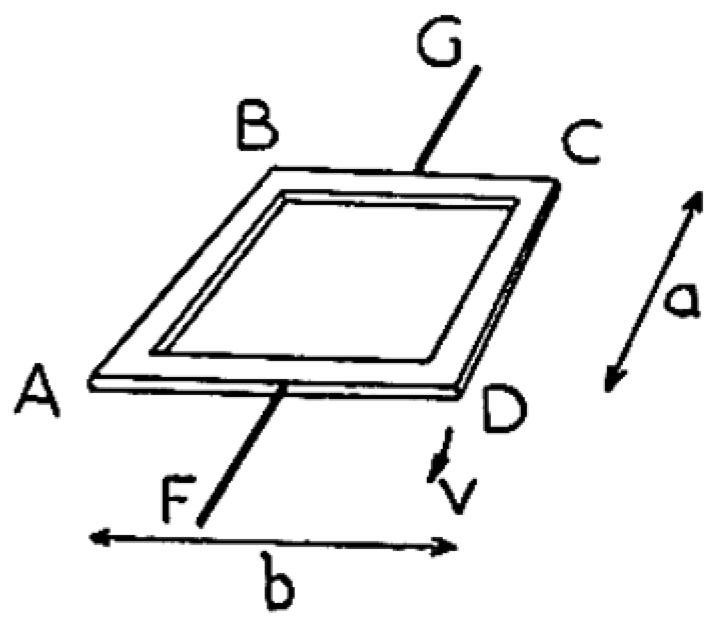
In the diagram ABCD represents the coil rotating about the axis FG. If the velocity with which AB and CD are rotating is v, the maximum rate of cutting of lines of force is $(2av\times 15)$ per second, where a is the length of AB. This must equal the E.M.F. induced in each turn of the coil in absolute electromagnetic units.

So for 20 turns, $\text{E.M.F.}=20\times 2av\times 15$ e.m.u.

But $v=5\times\pi\times b$

where b is the length of AD.

$$\therefore \quad \text{Maximum E.M.F.}=20\times 2\times 5\pi ab\times 15 \text{ e.m.u.}$$
$$=3000\pi\times ab$$
$$=3000\pi\times 800 \text{ e.m.u.}$$
$$=3000\pi\times 800\times 10^{-8} \text{ volt}$$
$$=\underline{0{\cdot}0754 \text{ volt}}$$



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